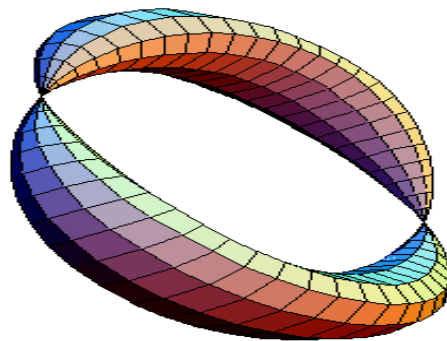# APPLIED MATHEMATICS HONOURS

Student Handbook 2022

School of Mathematics and Statistics

The University of Sydney

Contents

1 General Information

Honours in Applied Mathematics is a one-year program consisting of four 6 credit point units of study and 24 credit points of research project. For more details about the structure and completion requirements of Applied Mathematics honours program see Table A. You should start with a very useful general overview of honours in the School of Mathematics and Statistics and the Faculty of Science web page honours in Science.

1. **Entry requirements to honours programs**

   Honours is available to students who have a completed major in an area relevant to their project and have met the entry requirement to honours programs in the Faculty of Science. Notice that entry requirements to the honours program in Applied Mathematics vary slightly depending on whether the candidate is completing a Bachelor of Advanced Studies (Honours) degree or a Bachelor of Science (Honours) degree. Therefore, it is necessary to check whether you satisfy all entry requirements before applying for admission to honours in Applied Mathematics. See also more detailed guidelines for students at the University of Sydney willing to complete an honours year.

   For instance, admission to the Bachelor of Advanced Studies (Honours) requires:

   - completion of a bachelors degree from the University of Sydney or equivalent qualification from another tertiary institution in time to commence honours in the semester for which entry is sought;
   - completion of two majors or study deemed by the supervising faculty to be of comparable depth, of which at least one major is cognate to the honours area for which entry is sought;
   - a final Weighted Average Mark (WAM) of at least 65.00;
   - demonstrated evidence of contact with the School of Mathematics and Statistics, such as email correspondence with a prospective supervisor of a research project and the Honours Coordinator;
   - Expression of Interest (EOI) form signed-off by the Honours Coordinator.

   To qualify for admission to the Bachelor of Science (Honours), you must:

   - have qualified for or be a graduate with a Bachelor of Science degree from the University of Sydney or equivalent qualification from another tertiary institution;
   - have completed a relevant major (i.e. minimum of 24 credit points of 3000-level units of study) relating to the intended Honours discipline; have achieved a Science Weighted Average Mark (SCIWAM is the average over all second and third year units attempted) of at least 65.00 or have a credit average (65.00) in 48 credit points of relevant 2000-level and 3000-level units of study;
   - satisfy any additional criteria set by the relevant Head of School or Discipline;
   - demonstrate evidence of contact with the School of Mathematics and Statistics, such as email correspondence with a prospective supervisor of a research project and the Honours Coordinator;
   - submit Expression of Interest (EOI) form signed-off by the Honours Coordinator.

2. **Guidelines for students at the University of Sydney**

   – If you are currently enrolled in the Bachelor of Science and commenced your studies in 2018 or later (or transferred to the new curriculum version of your degree in 2018), how you commence honours will depend on how much of your degree you have completed.

   – If you have completed 96 credit points or less, we recommend you change to the combined Bachelor of Advanced Studies. You will then be able to undertake honours as an additional fourth year and graduate with two bachelor's degrees.

   – If you have completed 97–120 credit points and are on track to complete two majors, you have the choice to change to the combined Bachelor of Advanced Studies to complete honours or complete your Bachelor of Science then apply for the stand-alone Bachelor of Advanced Studies to undertake honours. You will need to apply before your penultimate semester.

   – If you have completed 121 credit points or more and are on track to complete two majors, you will need to complete your Bachelor of Science and apply for the stand-alone Bachelor of Advanced Studies to undertake honours. You will need to apply before your penultimate semester.

   – An appended honours degree, the Bachelor of Science (Honours), is an option for students who commenced their studies prior to 2018 or who have not completed or are not on track to complete two majors in their single bachelors degree.

3. **Project supervision**

   The candidate is required to find a prospective supervisor from among the Applied Mathematics staff, who is agreeable to supervise the candidate's project in the candidate's chosen topic. Students are required to submit the Expression of Interest form to the Honours Coordinator before submitting the honours application to the Faculty of Science.

4. **Students from other institutions**

   Students from institutions other than the University of Sydney must possess qualifications which are deemed equivalent to the above.

5. **Online honours applications to the Faculty of Science**

   Application and enrolment information should be obtained from the Faculty of Science from their website. For online applications to honours programs, see here. Application deadline for commencement in Semester 1 is 31 January 2022 and for commencement in Semester 2 is 30 June 2022.

6. **Honours scholarships**

   For scholarships available to honours students, see the website.

For further details, contact the Applied Mathematics honours coordinator

Prof Marek Rutkowski: Carslaw 814; marek.rutkowski@sydney.edu.au

2 Applied Mathematics Honours

Independent research can be a life changing experience. In this honours program you will complete a research project in the discipline of Applied Mathematics. Together with your supervisor, you will identify a novel research question and develop a model, or propose some mathematical or computational analysis. You will then carry out this program of work to produce results that can be interpreted in terms of the underlying real-world problem. Your work will be assessed by a twenty minute presentation towards the end of your honours year and a 40 to 60 page honours thesis. Successful completion of your honours will clearly demonstrate that you have mastered significant research and professional skills for either undertaking a PhD or any variety of future careers.

2.1 Honours structure

Honours in Applied Mathematics consists of four 6-credit point coursework units including two core units in Applied Computational Mathematics and Advanced Methods in Applied Mathematics. Students will also complete 24-credit points of research project.

2.2 Coursework (24 credit points)

1. **Core units (12 credit points)**
   Students are required to complete the following core units of study:

   – Semester 1: MATH 4411 Applied Computational Mathematics
   – Semester 2: MATH 4412 Advanced Methods in Applied Mathematics

2. **Selective units (12 credit points)**
   In addition to the core units, students should choose their two selective units from units offered by the School of Mathematics and Statistics listed as 4000 level or higher, which have not already been taken for credit with the proviso that at most one unit labelled 5000 or higher may be taken. For the full list of selective units available to students enrolled in Applied Mathematics honours, see Table A.

2.3 Honours project AMAT4103–4106 (24 credit points)

Each student is required to complete an honours project (composed of a written thesis and a seminar presentation) on an Applied Mathematics topic, under the supervision of a member of staff of the School of Mathematics and Statistics. Students should enrol in two project units in each semester: AMAT4103–4104 in Semester 1 and AMAT4105–4106 in Semester 2.

2.4 Applied Mathematics honours coordinator in 2022

Prof Marek Rutkowski
Room 814, Carslaw Building
Email: marek.rutkowski@sydney.edu.au

3 Core Units of Study

MATH4411 Applied Computational Mathematics *(Semester 1)*

Computational mathematics fulfils two distinct purposes within Mathematics. On the one hand the computer is a mathematicians laboratory in which to model problems too hard for analytical treatment and to test existing theories; on the other hand, computational needs both require and inspire the development of new mathematics. Computational methods are an essential part of the tool box of any mathematician. This unit will introduce you to a suite of computational methods and highlight the fruitful interplay between analytical understanding and computational practice.

In particular, you will learn both the theory and use of numerical methods to simulate partial differential equations, how numerical schemes determine the stability of your method and how to assure stability when simulating Hamiltonian systems, how to simulate stochastic differential equations, as well as modern approaches to distilling relevant information from data using machine learning. By doing this unit you will develop a broad knowledge of advanced methods and techniques in computational applied mathematics and know how to use these in practice. This will provide a strong foundation for research or further study.

Assessments: In this unit you will have to write reports for 3 computer assignments, each worth 20%. A final exam (worth 40%) will be on the mathematical theory behind the numerical algorithms you learn.

MATH4412 Advanced Methods in Applied Mathematics *(Semester 2)*

Much of our physical world is nonlinear. If you take two rulers and place one on top of another, the height of the combined object is the sum of the individual heights of each ruler. But whether you are looking at herds of bisons in a landscape, the viral load in an infective patients bloodstream, or the interaction of black holes far away in the universe, it turns out the sum of individual components does not necessarily give a true measure of reality. To describe these systems, we need methods that apply to nonlinear mathematical models.

This course will cover theoretical methods (some exact, some in limits and others that are qualitative) to describe, solve and predict the results of such models. Classical mathematical methods were developed for linear models. We will start with building blocks to describe models of semi-classical quantum mechanics and related orthogonal polynomials. These turn out to be generalizable to models that arise in modern physics, such as quantum gravity and random matrix theory. These lead naturally to integrable systems.

Assessments: Everyone will be expected to participate in discussions of exercises in tutorials. There will be two assignments, to be given out approximately in weeks 4 and 8, respectively. The exam is a take-home exam, to be done over a three-day period. Assessments: participation in tutorials: 5%; assignments 55%; final examination: 40%.

4   Important Dates in 2022

Semester 1: 21 February 2022 to 29 May 2022

- **Seminar presentation: Thursday and Friday in week 8**

- **Project submission: Monday, 23 May 2022 (week 13)**
  For students completing in Semester 1, 2022. An electronic file (pdf format) must be uploaded on Canvas before the deadline.

- **Examination period: 6–18 June 2022**

Semester 2: 1 August 2022 to 6 November 2022

- **Seminar presentation: Thursday and Friday in week 8**

- **Project submission: Monday, 31 October 2022 (week 13)**
  For students completing in Semester 2, 2022. An electronic file (pdf format) must be uploaded on Canvas before the deadline.

- **Examination period: 14–26 November 2022**

5   Final Honours Mark

The final honours mark SCIE4999 for each student is based on the following marking scheme:

- **50% for the project unit AMAT4103–4106,**

- **50% for four units of study (12.5% for each).**

The marking scale for Honours is significantly different from the undergraduate marking scale at the University of Sydney. The project will be marked with this scale in mind.

GRADE OF HONOURS	FACULTY-SCALE
First Class, with Medal	95–100
First Class (possibly with Medal)	90–94
First Class	80–89
Second Class, First Division	75–79
Second Class, Second Division	70–74
Third Class	65–69
Fail	00–64

6   Research Project in Applied Mathematics

A significant part of the honours year is the completion of a research project by each student. Each student must choose a project supervisor who is willing to supervise the student's chosen topic for the project. Project topics and supervisors should be finalised by the beginning of the first semester, so that students can commence work immediately on their projects. The following list shows the main Applied Mathematics research areas:

- Dynamical systems

- Geophysical and astrophysical fluid dynamics

- Industrial and biomedical modelling

- Integrable systems

- Mathematical biology

Members of the Applied Mathematics Research Group in the School of Mathematics and Statistics: Anna Aksamit, Eduardo Altmann, Holger Dullin, Jie Yen Fan, Ben Goldys, Georg Gottwald, Nalini Joshi, Peter Kim, Robert Marangell, Mary Myerscough, Milena Radnović, Marek Rutkowski, Sharon Stephen, Geoffrey Vasil, Martin Wechselberger, Zhou Zhou.

Their email addresses and research interests can be found here.

6.1   Project assessments

The written thesis will be marked by three examiners and each marking will therefore constitute 30% of the final mark from the project unit.

The final mark from the project unit AMAT4103–4106 will be awarded according to the following marking scheme:

- **90% for a written thesis,**

- **10% for a seminar presentation on the project.**

The seminar is an opportunity for each student to present the material of her/his research project to members of Applied Mathematics Research Group. The seminar talk will usually be of 25 minutes duration, with an additional 5 minutes set aside for questions. The presenter of the best talk will be awarded the Chris Cannon Prize. Marks for the thesis and seminar presentation will be awarded for:

(i)	selection and synthesis of source material;
(ii)	evidence of understanding;
(iii)	evidence of critical ability;
(iv)	clarity, style and presentation;
(v)	mathematical and/or modelling expertise.

6.2 Project guidelines

- The student should consult the supervisor on a regular basis, preferably at least once a week. This is the student's responsibility.

- A realistic schedule for work on the project should be drawn up at an early stage, and adhered to as closely as possible. If it proves necessary to modify the original plans, a revised schedule should be drawn up after discussion with the supervisor.

- At the end of Semester 1, a one page report is to be submitted to the Honours Coordinator. This report includes a half page description about the students aim/scope of the project and a half page description about what the student has achieved in Semester 1 and what the student wants to achieve in Semester 2. This report has to be approved by the supervisor before submission.

- The project should be based on some four to six original primary source articles, which themselves represent a substantial contribution to the topic. Secondary sources, such as books, review papers, etc., should also be consulted and cited.

- The thesis should be both a discursive and a critical account of the selected topic. It should be written at a level that an expert Applied Mathematician can be expected to understand. The work must contain substantial mathematical content.

- Students are recommended to use LaTeX in typesetting their projects. Additional information on LaTeX can be found here.

- The length of the written thesis should be between 40 to 60 typed (normal LaTeX font size) A4 pages. Only in exceptional circumstances, and after consultation with the supervisor, should the project exceed 60 pages. This number includes all figures, contents pages, tables, appendices, etc. Computer programs essential to the work should be included (with adequate commentary) as additional material.

- Computer programs essential to the work should be included (with adequate commentary) as additional material in appendices.

- Students should be careful to provide full and correct referencing to all material used in the preparation of projects. Be explicit in stating what is your contribution and what is someone else's contribution. Avoid quoting verbatim unless reinforcing an important point.

- Three examiners will be appointed to assess each written thesis. Although marking schemes may differ, the assessment of the thesis will be based on:
  - (i) selection and synthesis of source material;
  - (ii) evidence of understanding;
  - (iii) evidence of critical ability;
  - (iv) clarity, style and presentation;
  - (v) mathematical and/or modelling expertise.

- Students who have worked on their project topics as Vacation Scholars are required to make a declaration to that effect in the preface of their thesis.

6.3 Topics for the project and supervisors

You will find below a list of possible project topics for honours students in Applied Mathematics in 2022. Prospective students interested in any of these topics are encouraged to discuss them with the named supervisors as early as possible. The list is not exhaustive and thus you may wish to suggest your own topic for project or discuss any other topic with a potential supervisor. Notice that each student must find a member of staff who will agree to supervise the project before applying for admission to Applied Mathematics honours.

Complex networks and social-media data
Prof E. Altmann; Carslaw 523; eduardo.altmann@sydney.edu.au; phone 9351-4533

Please contact me if you are interested in projects combining data analysis and mechanistic models of complex networks and social-media data (time series and natural language processing).

Monte Carlo methods in triangulation problems
Prof E. Altmann; Carslaw 523; eduardo.altmann@sydney.edu.au; phone 9351-4533

The goal of this project is to investigate how Markov Chain Monte Carlo methods can be used to obtain efficient triangulation of manifolds in different dimensions. After reviewing the known results for simple configurations (in low dimensions), we will focus on computational methods to efficiently find triangulations. Within a Monte Carlo framework, we will investigate the efficiency of different proposal steps such as moves that merge and create triangles. This project involves programming, it lies in the intersection between Applied and Pure mathematics, and will be co-supervised by Dr. Jonathan Spreer and Prof. Eduardo Altmann. Related work:

[1] T. Aste, R. Gramatica, and T. Di Matteo: Random and Frozen States in Complex Triangulations. *Philosophical Magazine*, 92:1-3 (2012), 246–254.

Predictability of epidemic-spreading models
Prof E. Altmann; Carslaw 523; eduardo.altmann@sydney.edu.au; phone 9351-4533

The aim of this project is to quantify in which extent the spreading of an epidemic can be forecasted in advance. Prediction of the spreading is limited due to the combination of random fluctuations, unknown information, and the non-linear character of the underlying dynamics. The focus of this project will be on predicting the peak of an infection [1,2]. It will involve analytical and numerical investigations of ODE models and data analysis of time series of number of infections in different geographical areas.

[1] M. Castro, S. Ares, J. A. Cuesta, S. Manrubia: The turning point and end of an expanding epidemic cannot be precisely forecast. *Proc. Natl. Acad. Sci. U.S.A.* 117/46 (2020) 26190–26196.

[2] Claus O. Wilke and Carl T. Bergstrom Predicting an epidemic trajectory is difficult. *Proc. Natl. Acad. Sci. U.S.A.* 117/46 (2020) 28549–28551.

The tennis racket effect

Prof H. Dullin; Carslaw 714; holger.dullin@sydney.edu.au; phone 9351-4083

The tennis racket effect is the observation that when you throw a tennis racket in the air (initially holding it at its handle and facing the strings) and catch its handle after one revolution, then usually the racket also flips by 180 degrees about its long axis, so that in the end you are looking at the opposite side of the strings. Part of the explanation is that a rotation of a free rigid body about its middle axis of inertia is unstable. This does now, however, explain why the rotation around the long axis is somewhat synchronised. Explanations for this effect have been proposed, but appear to either miss the point or to be too complicated. The goal of the project is to use the geometric phase to derive at a simple quantitative explanation of the effect, and to verify it numerically.

Chaotic dynamics of the pentagon

Prof H. Dullin; Carslaw 714; holger.dullin@sydney.edu.au; phone 9351-4083

A chain of planar rigid bodies is a simple mechanical system with n segments connected by joints that allow free rotation. Connecting the first segment to the last by another joint gives a closed chain. Since the distance between the joints is fixed the closed chain has n degrees of freedom. Reduction by translations and rotations leaves n 3 degrees of freedom specifying the shape. For certain parameters the dynamics of this system is chaotic in the sense of Anosov. The goal of the project is to study the dynamics in the first non-trivial case of the pentagon (n=5). In a chaotic system periodic orbits are dense in phase space, and the goal of the project is to find and describe the periodic orbits of this system, using a combination of analytical and numerical tools.

Non-normality in the Hopf bifurcation

Prof H. Dullin; Carslaw 714; holger.dullin@sydney.edu.au; phone 9351-4083

A real square matrix is normal if it commutes with its transpose. For example, orthogonal, symmetric, and skew-symmetric matrices are normal matrices. Non-normal matrices are important in regards to understanding stability and instability in dynamical systems. For example, certain types of instabilities in fluid dynamics can be explained using non-normal matrices. The Hamiltonian Hopf bifurcation describes the bifurcations that can occur when two pairs of imaginary eigenvalues collide on the imaginary axis and branch off into the complex plane. Non-normal matrices appear naturally near this bifurcation. The goal of the project is to describe and analyse the transient growth associated to non-normality as it appears near this bifurcation.

Symplectic integration of the regularised planar circular restricted 3 body problem

Prof H. Dullin; Carslaw 714; holger.dullin@sydney.edu.au; phone 9351-4083

The restricted three body problem describes the motion of a test particle in the field of two heavy masses rotating around each other in circular orbits. The problem has a singularity

when the test particle collides with either of the other masses. The collision can be regularised, such that the solutions are defined for all times. The goal of the project is to construct and implement a symplectic integration method for the regularised problem. This can then be used to study periodic orbits in this chaotic system, in particular collision orbits. The study of collision orbits is interesting and relevant because double collision orbits describe the motion of a spacecraft from one body to the other.

Data-driven modelling: Finding models for observations in finance and climate
Prof G. Gottwald; Carslaw 625; georg.gottwald@sydney.edu.au; phone 9351-5784

When given data, which may come from observations of some natural process or data collected form the stock market, it is a formidable challenge to find a model describing those data. If the data were generated by some complex dynamical system one may try and model them as some diffusion process. The challenge is that even if we know that the data can be diffusive, it is by no means clear on what manifold the diffusion takes place. This project aims at applying novel state-of-the-art methods such as diffusion maps and nonlinear Laplacian spectral analysis to determine probabilistic models. You will be using data from ice cores encoding the global climate of the past 800kyrs as well as financial data. In the latter case you might be able o recover the famous Black-Scholes formula (but probably not). This project requires new creative ideas and good programming skills.

Optimal power grid networks and synchronisation
Prof G. Gottwald; Carslaw 625; georg.gottwald@sydney.edu.au; phone 9351-5784

Complex networks of coupled oscillators are used to model systems from pacemaker cells to power grids. Given their sheer size we need methods to reduce the complexity while retaining the essential dynamical information. Recent new mathematical methodology was developed to describe the collective behaviour of large networks of oscillators with only a few parameters which we call "collective coordinates." This allows for the quantitative description of finite-size networks as well as chaotic dynamics, which are both out of reach for the usually employed model reduction methods.

You will apply this methodology to understand causes of and ways to prevent glitches and failure in the emerging modern decentralised power grids. As modern societies increase the share of renewable energies in power generation the resulting power grid becomes increasingly decentralised. Rather than providing a power supply constant in time, the modern decentralised grid generates fluctuating and intermittent supply. It is of paramount importance for a reliable supply of electric power to understand the dynamic stability of these power grids and how instabilities might emerge. A reliable power-grid consists of well-synchronised power generators. Failing to assure the synchronised state results in large power outages as, for example, in North America in 2003, Europe in 2006, Brazil in 2009 and India in 2012 where initially localised outages cascade through the grid on a nation-wide scale. Such cascading effects are tightly linked to the network topology. Modern power-grids face an intrinsic challenge: on the one hand decentralisation was shown to favour synchronisation in power grids, on the other hand decentralised grids are more susceptible to dynamic perturbations such as intermittent power supply or overload.

The project uses analytical methods as well as computational simulation of models for power grids. You will start with a simple network topology and then, if progress is made, use actual power grid topologies.

Data assimilation in numerical weather forecasting and climate science
Prof G. Gottwald; Carslaw 625; georg.gottwald@sydney.edu.au; phone 9351-5784

Data assimilation is the procedure in numerical weather forecasting whereby the information of noisy observations and of an imperfect model forecast with chaotic dynamics, we cannot trust, is combined to find the optimal estimate of the current state of the atmosphere (and ocean). Data assimilation is arguably the most computationally costly step in producing modern weather forecasts and has been topic of intense research in the last decade. There exists several approaches, each of which with their own advantage and disadvantages. Recently a method was introduced to adaptively pick the best method to perform data assimilation. This method employs a switch which, although it seems to work, has not been linked to any theoretical nor physical properties of the actual flow. This project will be using toy models for the atmosphere to understand the witch with the aim of improving the choice of the switching parameter.

Networks of coupled oscillators
Prof G. Gottwald; Carslaw 625; georg.gottwald@sydney.edu.au; phone 9351-5784

Many biological systems are structured as a network. Examples range from microscopic systems such as genes and cells, to macroscopic systems such as fireflies or even an applauding audience at a concert. Of paramount importance is the topography of such a network, i.e., how the nodes, let's say the fireflies, are connected and how they couple. Can they only see their nearest neighbours, or all of them. Are some fireflies brighter than others, and how would that affect the overall behaviour of a whole swarm of fireflies? For example, the famous 'only 6 degrees of separation'-law for the connectivity of human relationships is important in this context.

In this project we aim to understand the influence of the topography of such a network. Question such as: How should a network be constructed to allow for maximal synchronization will be addressed. This project requires new creative ideas and good programming skills.

Discrete soliton equations
Prof N. Joshi; Carslaw 629; nalini.joshi@sydney.edu.au; phone 9351-2172

Famous PDEs such as the Korteweg-de Vries equation (which have soliton solutions) have discrete versions (which also have soliton solutions). These discrete versions are equations fitted together in a self-consistent way on a square, a 3-cube or an N-dimensional cube. These have simple, beautiful geometric structures that provide information about many properties: solutions, reductions to discrete versions of famous ODEs, and deeper aspects such as Lagrangians. This project would consider generalisations of such structures and/or properties of the solutions, such as finding their zeroes or poles.

Integrable discrete or difference equations

Prof N. Joshi; Carslaw 629; nalini.joshi@sydney.edu.au; phone 9351-2172

The field of integrable difference equations is only about 20 years old, but has already caused great interest amongst physicists (in the theory of random matrices, string theory, or quantum gravity) and mathematicians (in the theory of orthogonal polynomials and soliton theory). For each integrable differential equation there are, in principle, an infinite number of discrete versions. An essay in this area would provide a critical survey of the many known difference versions of the classical Painlevé equations, comparisons between them, and analyse differing evidence for their integrability. Project topics would include the derivation of new evidence for integrability. The field is so new that many achievable calculations remain to be done: including derivations of exact solutions and transformations for the discrete Painlevé equations.

Exponential asymptotics

Prof N. Joshi; Carslaw 629; nalini.joshi@sydney.edu.au; phone 9351-2172

Near an irregular singular point of a differential equation, the solutions usually have divergent series expansions. Although these can be 'summed' in some way to make sense as approximations to the solutions, they do not provide a unique way of identifying a solution. There is a hidden free parameter which has an effect like the butterfly in chaos theory. This problem has been well studied for many classes of nonlinear ODEs but almost nothing is known for PDEs and not much more is known for difference equations. This project would include studies of a model PDE, like the famous Korteweg-de Vries equation near infinity, or a difference equation like the string equation that arises in 2D quantum gravity.

Cellular automata

Prof N. Joshi; Carslaw 629; nalini.joshi@sydney.edu.au; phone 9351-2172

Cellular automata are mathematical models based on very simple rules, which have an ability to reproduce very complicated phenomena. (If you have played the Game of Life on a computer, then you have already seen automata with complicated behaviours.) This project is concerned with the mathematical analysis of their solutions, which lags far behind corresponding developments for differential or difference equations.

In this project, we will consider a family of cellular automata called parity filter rules, for which initial data are given on an infinite set. For example, consider an infinitely long train of boxes, a finite number of which have a ball inside, whilst the remainder are empty. At each time step, there is a simple rule for moving the leftmost ball in a box to the next empty box on the right. Continue until you have finished updating all nonempty boxes in the initial train. (Try this out for yourself with adjacent boxes with three balls, followed by two empty boxes and then two boxes with balls inside. What do you see after one update? Two updates?) It turns out that these box-and-ball systems replicate solitons, observed in solutions of integrable nonlinear PDEs. In this project, we will consider how to derive parity filter rules from nonlinear difference equations, and how to analyse their solutions. One direction for the project is to analyse the solutions as functions of initial data. Another direction is to develop ways to describe long-term behaviours.

Modelling the evolution of human post-menopausal longevity and pair bonding

Prof P. Kim; Carslaw 621; peter.kim@sydney.edu.au; phone 9351-2970

A striking contrast between humans and primates is that human lifespans extend well beyond the end of the female reproductive years. Natural selection favours individuals with the greatest number of offspring, so the presence of a long female post-fertile period presents a challenge for understanding human evolution.

One prevailing theory that attempts to explain this paradox proposes that increased longevity resulted from the advent of grandmother care of grandchildren. We have developed preliminary age-structured PDE models and agent-based models to consider the intergenerational care of young proposed by this Grandmother Hypothesis. The project will involve extending the models to consider whether the presence of grandmothering could increase the optimum human longevity while simultaneously maintaining a relatively early end of fertility as seen in humans (and killer whales).

Analytical approaches will involve developing numerical schemes for the PDEs and analytically and numerically studying the steady state age distributions and growth rates of the populations with and without grandmothering and under different life history parameters, e.g. longevity and end of fertility.

We have now also begun to explore mating strategies, especially pair bonding, yet another unique human characteristic among mammals. Speculations about how pair bonding developed from our ancestral roots abound and are open to being quantified, modelled, and analysed. Like the grandmothering models, these investigations will involve PDEs or agent-based models.

Modelling cancer immunotherapy

Prof P. Kim; Carslaw 621; peter.kim@sydney.edu.au; phone 9351-2970

A next generation approach to treating cancer focuses on cancer immunology, specifically directing a person's immune system to fight tumours. Recent directions in cancer immunotherapy include

- Oncolytic virotherapy: infecting tumours with genetically-engineered viruses that preferentially destroy tumour cells and induce a local anti-tumour immune response,

- Preventative or therapeutic cancer vaccines: stimulating a person's immune system to attack tumour colonies to prevent or hinder tumour development,

- Cytokine therapy: using immunostimulatory cytokines to recruit immune cells and enhance existing anti-tumour immune responses.

These treatments can be used alone or in combination with each other or with other forms of treatment such as chemotherapy. Since immunotherapy often involves immune responses against small tumours, often close to inception, they are highly spatially dependent and often probabilistic. The goal of the will be to develop differential equation and possibly probabilistic agent-based models to understand the tumour-virus-immune dynamics around a small,

developing tumour and determine conditions that could lead to effective tumour reduction or complete elimination. The project will involve developing the models and schemes for numerically simulating the ODE and PDE systems, and if possible, performing a stability analysis of the ODE system.

Geometric aspects of Turing bifurcations
Dr R. Marangell; Carslaw 720; robert.marangell@sydney.edu.au; phone 9351-5795

The Turing bifurcation is a classical example of a diffusion driven instability. This project will attempt to look at some geometric aspects of the system at the onset of a Turing bifurcation, as well as potential factorisation of the system when the diffusion is either extremely large or extremely small.

Fast-slow splitting in characteristic determinants
Dr R. Marangell; Carslaw 720; robert.marangell@sydney.edu.au; phone 9351-5795

Eigenvalue problems on a finite interval can often be characterised in terms of the vanishing of the determinant of a matrix. Such a determinant is called the characteristic determinant of the system. When multiple time-scales are present, this often results in the ability to factor the characteristic determinant into characteristic determinants of lower-dimensional systems. This project will look at how this factorisation takes place, based on the entries of the original system.

Symmetries in PDEs
Dr R. Marangell; Carslaw 720; robert.marangell@sydney.edu.au; phone 9351-5795

This project is about the relationship between symmetries of partial differential equation, the coordinate systems in which the equation admits solutions via separation of variables and the properties of the special functions that arise in this manner. A major focus of this project lying at the intersection of geometry, algebra and analysis is the characterisation of separable coordinate systems in terms of the second-order symmetry operators of for the equations.

Chemotaxis in models with zero/negative diffusivity
Dr R. Marangell; Carslaw 720; robert.marangell@sydney.edu.au; phone 9351-5795

Chemotaxis is the movement of a cell via advection either towards or away from a chemical source. It has been used in many biological models, from slime-moulds to motile bacteria, to roadway construction by humans. Typically linear diffusivity has been studied, but lately models where the diffusivity is allowed to change sign have become of interest. This project will examine the existence of travelling wave solutions in such models, as well as some elementary stability properties of such solutions.

Stability in a model of herd grazing and chemotaxis

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This project will examine a model of the formation of a herd of grazing animals. The model will focus on two major factors, how the animal seeks food and how the the animals interact with each other. Remarkably, the model shares many properties with another, well studied model, that of so-called bacterial chemotaxis. The aim of this project will be to analyse, both numerically and analytically, such a model, and to understand certain special solutions in the model, called travelling waves, as well as their stability.

Absolute spectrum of St. Venant roll waves

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Roll waves are a phenomenon that occurs when shallow water flows down an inclined ramp. Mathematically they can be modelled by the St. Venant equations. Typically roll waves occur as periodic solutions, however if they are far enough apart, they can be treated as solitary waves. In this case, the spectrum of the linearised operator governs their dynamics, and in particular, their stability properties. This project will focus on computing the absolute and essential spectrum of these solitary waves. Medium computational skills are required for this project.

Other possible research topics

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Other projects under the supervision of Dr Marangell include topics in the areas of non-linear standing or travelling waves, topics in the application of geometric and topological methods in dynamical systems and PDEs, symmetries in ODEs and PDEs and other research topics in the history of mathematics and science in general. Examples of nonlinear standing and travelling waves come from models in a wide range of areas which include mathematical biology, chemistry and physics. More specific examples would be standing/travelling waves in population dynamics, combustion models, and quantum computing, but really there are many, many examples, so please contact Dr Marangell for further details.

PDE models for the distribution of ingested lipids in macrophages in atherosclerotic plaques

Prof M. R. Myerscough; Carslaw 626; mary.myerscough@sydney.edu.au; phone 9351-3724

Atherosclerotic plaques are accumulations of lipid (fat) loaded cells and necrotic (dead) cellular debris in artery walls. They are caused by LDL (which carries 'bad cholesterol') penetrating the blood vessel wall, becoming chemically modified (usually oxidised) and setting off an immune reaction. In response to this immune reaction, macrophages (a type of white blood cell) enter the artery wall and consume the modified LDL.

In this way macrophages accumulate lipids and as more LDL and more cells enter the vessel walls the population of cells also grows. Other processes can affect the growth or regression of the plaque, such as cell death, cells leaving the tissue and lipid export from inside cells to HDL (which carries 'good cholesterol' which is good because it's been carried away from the plaque). When atherosclerotic plaques grow very large and rupture they can cause heart attacks and strokes which are one of the two leading causes of death in the developed world. (The other is cancer.) We have written a partial differential equation model for the accumulation of cells and lipids in plaques. In this model, the number of macrophages in the plaque is a function of both time t and accumulated lipid a. The primary equation is an advection equation with nonlinear source and sink terms, including a term with an integral convolution that models what happens when macrophages phagocytose (=eat) other macrophages that are dead or dying. We have done an analysis of this model at steady state when all the processes (lipid ingestion, macrophages leaving the plaque, the action of HDL) occur at a constant rate. This project will build on this analysis and has the aim of producing numerical solutions to the model when model processes are functions of a, the accumulated lipid inside the cell. This project is particularly suitable for students who are interested in applications of mathematics to biomedical problems, have completed a third year unit on PDEs and have at least some experience in coding in Matlab, C, Python or similar.

PDE models for real and fake macrophages in atherosclerotic plaques
Dr M. Watson; Carslaw 534 and Prof M. R. Myerscough; Carslaw 626; phone 9351-3724

Atherosclerosis, a major cause of heart attacks and strokes, involves the growth of fatty, cellular plaques in the walls of arteries. In these plaques, the main source of fat (lipid) is LDL, which enters the artery wall from the bloodstream, and the two main cell types are macrophages (a type of immune cell) and smooth muscle cells (SMCs; mural cells native to the artery). Macrophages and SMCs have long been thought to have distinct and well-defined roles: macrophages work to find, consume and remove lipid from the plaque, while SMCs generate a cap of tissue that isolates the (dangerous) content of the plaque from the bloodstream. Recent advancements in fate and lineage tracing of plaque cells have, however, completely changed this paradigm. It is now known that many cells previously thought to be real macrophages are, in fact, SMCs that have left the cap and adopted a macrophage-like phenotype (fake macrophages). Evidence suggests that these SMC-derived macrophages consume and remove lipid less effectively than real macrophages, but little is known about how fake macrophages influence plaque progression. We have developed a preliminary PDE model for the accumulation of macrophages and lipids in atherosclerotic plaques. We consider two separate macrophage populations (real and fake), where each population is a function of both time t and accumulated lipid a. As the two cell populations compete for (and recycle) the same pool of lipid, the dynamics of these populations are intrinsically coupled. The cell populations are modelled by advection equations with nonlinear source and sink terms, including non-local terms that model jumps in internalised lipid content when a cell divides or consumes a dying cell (efferocytosis). In this project, we will use our preliminary model to investigate how fake macrophages, which behave quite differently from real macrophages, can alter plaque dynamics. Should we find that fake macrophages negatively impact plaque progression, we will then numerically investigate potential treatment strategies to mitigate these effects. This project will be suitable for students who are interested in the application of mathematical modelling to cutting-edge biomedical science, and who have completed third year units on PDEs and/or numerical computation.

Mathematical billiards

A/Prof M. Radnović; Carslaw 624; milena.radnovic@sydney.edu.au; phone 9351-5782

Mathematical billiards have been an established topic for research for about one century. They have application in any situation involving collisions and reflections. They are used as a model for the popular game of billiards, and also in laser techniques, the statistical interpretation of the second law of thermodynamics wind-tree model, the dynamics of ideal gas, tri-atomic chemical reactions etc. The field of mathematical billiards is at the cutting edge of mathematics research, and work in the field is highly valued: several Fields Medals were recently awarded for contributions in the area. The research on this project can vary from making computer simulations to more theoretical work. Writing an essay is also available.

Poncelet porisms

A/Prof M. Radnović; Carslaw 624; milena.radnovic@sydney.edu.au; phone 9351-5782

Suppose that two conics are given in the plane, together with a closed polygonal line inscribed in one of them and circumscribed about the other one. The Poncelet porism states that then infinitely many such closed polygonal lines exist and all of them with the same number of sides. That statement is one of most beautiful and deepest contributions of the 19th century geometry and has many generalisations and interpretations in various branches of mathematics. In this essay, the student will present rich history and current developments of the Poncelet porism.

Elliptical billiards and their periodic trajectories

A/Prof M. Radnović; Carslaw 624; milena.radnovic@sydney.edu.au; phone 9351-5782

We consider billiards in a domain bounded by arcs of several conics belonging to a confocal family. When the boundary of such a billiard does not contain reflex angles, the system turns out to be integrable. Geometrically, the integrability has the following manifestation - for each billiard trajectory, there is a curve, called caustic, which is touching each segment of the trajectory. For elliptical billiards, the caustics are conics from the same confocal family. Integrability implies that the trajectories sharing the same caustic are either all periodic with the same period or all non-periodic.

On the other hand, if there is at least one reflex angle on the boundary, the integrability will be broken, although the caustics still exist. Such billiards are thus called pseudo-integrable and there may exist trajectories which are non-periodic and periodic with different periods sharing the same caustic.

An essay on this topic would provide a review of classical and modern results related to the elliptical billiards. In a project, the student would explore examples of billiard desks.

Control of boundary-layer flows

A/Prof S. Stephen; Carslaw 525; sharon.stephen@sydney.edu.au; phone 9351-3048

This project is in the field of hydrodynamic stability of boundary-layer flows where viscous effects are important. The aim is towards understanding more fully the transition process from a laminar flow to a turbulent one.

We will consider rotating flows which are relevant to the flow over a swept wing and to rotor-stator systems in a turbine engine. Experiments show that the boundary layer becomes unstable to stationary or travelling spiral vortices.

The project will investigate the effect of different surface boundary conditions on boundary-layer flows over rotating bodies. Effects such as suction, partial-slip, compliance and wall shape can be modelled. Suction, for example, is used to achieve laminar flow control on swept wings.

The resulting system of governing ordinary differential equations will be solved numerically for the basic flow, determining important values such as the wall shear. The linear stability of these flows to crossflow instabilities will be investigated. These take the form of co-rotating vortices, observed in experiments, and only occur in three-dimensional boundary layers.

The flow for large Reynolds number, corresponding to large values of rotation, will be considered. In this case the boundary layer thickness will be very small so asymptotic methods of solution will be used. Different asymptotic regimes will need to be considered and solutions obtained in each region. Matching the solutions between the regimes and satisfying the boundary conditions will lead to an eigenrelation. Inviscid and viscous instability modes will be considered.

The effect of the surface boundary conditions on the disturbance wave number and wave angle will be determined. This will have applications in possible control of boundary layers as boundaries causing stabilisation of the instabilities could lead to a delay in the transition process from a laminar flow to a turbulent flow.

Probing the universe with nonlinear echoes

A/Prof L. Tzou; Carslaw 613; leo.tzou@sydney.edu.au; phone 9351-3114

We see because electromagnetic waves stimulate our eyes. Some animals, however, see other types of waves - bats generate sound waves, using the echoes to pinpoint the size and location of their prey.

Although human ingenuity has also given us some tools to see beyond our eyes, present tomography methods nevertheless face limitations - seismology often suffers from a scarcity of detectors; telescopes can collectively receive only a tiny fraction of the signals propagating through the universe. These problems effectively require us to deduce large-scaled information from a single measurement point, inspiring challenging mathematical models that we have yet to overcome.

In this project the student will learn the necessary mathematics to potentially participate on my current work on single detector gravitational wave astronomy.

Non-equilibrium statistical mechanics of a chain: the birth of temperature

Dr G. Vasil; Carslaw 627; geoffrey.vasil@sydney.edu.au; phone 9351-4163

If you release a simple, frictionless pendulum from a given height, it will oscillate forever in periodic motion. If you attach a second pendulum at the bottom of the first, the result becomes chaotic with all the associated rich behaviour. But the system will still more-or-less swing back and forth forever.

If you attach 100, 1,000, or 1,000,000 more frictionless elements to the bottom of the configuration something even more interesting happens. The swinging motion begins to go away. The initial coherent kinetic energy transfers into the small-scale random fluctuations of the individual elements. The transfer of energy is a process called "thermalisation" (or the birth of temperature), and the damping of the motion is the emergence of friction in an otherwise dissipation-free system. Even more fascinating, the system can generate configurational entropy, and this gives rise to emergent entropic forces. This is a fancy way of saying that the initial inextensible pendulum chain configuration turns into a hot bouncing bungee cord.

How does this happen exactly? How fast does temperature, entropy, friction, and tension emerge? What can we predict about the statistics? What happens if we drive the system externally? What does this say about hydrodynamical turbulence? This project will investigate all any and all of these question and more. The project will start by directly simulating the pendulum system numerically. After analysing the results, we can use theoretical techniques from equilibrium and non-equilibrium statistical mechanics to better understand these kinds of systems.

Computational projects with Dedalus

Dr G. Vasil; Carslaw 627; geoffrey.vasil@sydney.edu.au; phone 9351-4163

Dedalus is a multi-purpose solver than can simulate a broad array of nonlinear partial differential equations (PDEs). You can find out about Dedalus here: http://dedalus-project.org

I have been leading the development of this project for the past few years. The idea behind Dedalus is that the user can enter their equations in text-based format and experiment with different behaviour much more easily than anything that came before. The code runs on computers ranging from laptops to the world's most powerful supercomputers. You can see some examples of past work done here: http://vimeo.com/dedlaus

In the past, my collaborators and I have solved problems ranging from solar magnetohydrodynamics, the dynamics of boats in Arctic seas, buckling elastic beams, nonlinear optics on topologically complex graphs, turbulent thermal convection, nuclear burning flames in white dwarf stars, cancer modelling, and more.

I have a range of potential projects that students can investigate with Dedalus. Some of these are extensions of past problems, and some are entirely new. I'm also happy to talk to students who have a particular application in mind that they would like to try. The idea is to simulate something interesting and model the behaviour after you know the answer!

Improved numerical algorithms for multidimensional spectral methods
Dr G. Vasil; Carslaw 627; geoffrey.vasil@sydney.edu.au; phone 9351-4163

Simulating important physical, biological, finical, and mathematical dynamics usually involves solving a large system of linear equations. An important key to many numerical methods is finding a way to solve linear systems in as few operations as possible. The first way to do this is to use a basis that produces matrices with as few non-zero elements as possible. But even with very sparse matrices, then actual time to find the solution can still differ by many orders of magnitude with different methods.

Two general classes of methods exist for solving large, sparse, linear systems. The first class is "direct methods." These are things like Gaussian elimination that you commonly learn in undergraduate courses. The second class are "iterative methods." Iteration involves applying a matrix to a vector over and over again in a clever way that converges to the desired solution. Both of these methods have their pros and cons.

The idea of this project is to study iterative methods in a new context. This project has the potential to accelerate important computational algorithms by many orders of magnitude. The results of this would contribute back to the code base of the Dedalus project and apply to many different areas.

Geometric singular perturbation theory and its applications
Prof M. Wechselberger; Carslaw 628; martin.wechselberger@sydney.edu.au; phone 9351-3860

Projects under the supervision of Prof M. Wechselberger include research topics in the field of dynamical systems with an emphasis on the study of pattern generation of so called multiple time-scales dynamical systems. These multi-scale systems are ubiquitous in nature and control most of our physiological rhythms. For instance, one cycle of a heartbeat consists of a long interval of quasi steady state interspersed by a very fast change of state, the beat itself. The same is true for the creation of neural action potentials. In these physiological systems, the very fast relaxation of energy leads to the notion of a relaxation oscillator and indicates physiological processes evolving on multiple timescales.

Topics could range from a theoretical study of possible multiple time-scales dynamics associated with relaxation oscillators to the analysis of a concrete physiological rhythm and algorithmic implementation of geometric singular perturbation theory.

For more information on possible topics, please have a look at, e.g., the monograph *Geometric singular perturbation theory beyond the standard form* by Prof M. Wechselberger:

7 Prizes and Awards

The following prizes may be awarded to Applied Mathematics Honours students of sufficient merit. Students do not need to apply for these prizes, which are awarded automatically.
A complete list of the prizes and scholarships offered by the School of Mathematics and Statistics can be found here.

Joye Prize in Mathematics   Value: **$5300, with medal and shield**

Awarded to the most outstanding student completing Honours in the School of Mathematics and Statistics.

University Medal

Awarded to Honours students who perform outstandingly. The award is subject to Faculty rules, which require a Faculty mark of 90 or more in Applied Mathematics Honours and a Third Year WAM of 80 or higher. A medal is always awarded when the Faculty mark is 95 or higher. More than one medal may be awarded in any year.

K. E. Bullen Memorial Prize   Value: **$1000**

Awarded on the recommendation of the Head of the School of Mathematics and Statistics in consultation with the professors of Applied Mathematics to the most proficient student in Applied Mathematics Honours, provided that the student's work is of sufficient merit.

Barker Prize   Value: **$550**

Awarded at the Fourth (Honours) Year examination for proficiency in Pure Mathematics, Applied Mathematics or Mathematical Statistics.

M. J. and M. Ashby Prize   Value: **$360**

Offered for the best project, submitted by a student in the Faculty of Science, that forms part of the requirements of Honours in Pure Mathematics, Applied Mathematics or Mathematical Statistics.

Norbert Quirk Prize No IV   Value: **$250**

Awarded for the best project on a given mathematical subject by a student enrolled in a Fourth Year course in Mathematics (Pure Mathematics, Applied Mathematics or Mathematical Statistics) provided that the essay is of sufficient merit.

Australian Federation of Graduate Women Prize in Mathematics.   Value: **$175**

Awarded on the recommendation of the Head of the School of Mathematics and Statistics, to the most distinguished woman candidate for the degree of BA or BSc who graduates with first class Honours in Applied Mathematics, Pure Mathematics or Mathematical Statistics.

Chris Cannon Prize   Value: **$100**

For the best adjudged project seminar presentation of an Applied Mathematics Honours student.

8   AMSI Courses

Students are welcomed to check the courses offered in January 2022 at the

- AMSI Summer School

as well as the courses available via the

- Advanced Collaborative Environment (ACE).

In principle, at most one AMSI/ACE course can be taken for credit by enrolling in the unit AMSI4001. It should be noted, however, that this is only possible if very special circumstances can be demonstrated. In particular, it is not enough to show that a given AMSI/ACE course is beneficial for a student since AMSI/ACE can be completed without enrolment in AMSI4001. Furthermore, it should be stressed that enrolment in AMSI4001 can only be done in consultation with the student's supervisor and with explicit prior approvals by the Applied Mathematics honours coordinator (Prof M. Rutkowski) and the Schools honours coordinator (Prof L. Paunescu).

9   Rights and Responsibilities

Applied Mathematics Honours students will have access to the following:

- Office space and a desk in the Carslaw building.
- A computer account with access to e-mail, as well as LaTeX and printing facilities for the preparation of projects.
- A photocopying account paid by the School for assembling project source material.
- After-hours access to the Carslaw building.
- A pigeon-hole in room 728.
- Participation in the School's social events.
- Class representative at School meetings.

Applied Mathematics Honours students have the following obligations:

- Regular attendance at the weekly seminars in Applied Mathematics.

- Have regular meetings with project supervisors, and meet all deadlines.

- Utilise all School resources in an ethical manner.

- Contribute towards the academic life in Applied Mathematics at the School of Mathematics and Statistics.

10   Life After Fourth Year

Postgraduate Studies

Many students completing the Honours programme have in the past gone on to pursue postgraduate studies at the University of Sydney, at other Australian universities, and at overseas universities. Please see the School's Coordinator of Postgraduate Studies if interested in enrolling for an MPhil or PhD at the School of Mathematics and Statistics. Students who do well in Applied Mathematics Honours may be eligible for postgraduate scholarships, which provide financial support during subsequent study for higher degrees at Australian universities. The honours coordinator is available to discuss options and provide advice to students interested in pursuing studies at other universities.

Careers

Students seeking assistance with post-grad opportunities and job applications should feel free to ask lecturers most familiar with their work for advice and written references. The Director of the Applied Mathematics Teaching Program and Course Coordinators may also provide advice and personal references for interested students.