Assignment 2

"Curves"

Due Thursday, 18 August, at 16:00 in the assignment box for MATH3405. The box is located on **Level 4** in the Mathematics (Priestley) building (67). It is number 035 in the **brown lot of boxes** (there are two lots, to find ours, turn right as you come from the stairs).

Please use a cover sheet!

Clearly state your assumptions and conclusions, and justify all steps in your work. Marks will be deducted for sloppy or incomplete working.

Q1 (Length)
Let $c \in \mathbb{R} \setminus \{0\}$, and define $\alpha\colon \mathbb{R} \to \mathbb{R}^2$ by

$$t \mapsto (e^{ct}\cos t, e^{ct}\sin t).$$

(a) Determine $\text{length}_{[a,b]}(\alpha)$ for any $a, b \in \mathbb{R}$ with $a \le b$.
(b) Determine each of $\text{length}_{\mathbb{R}}(\alpha)$, $\text{length}_{(-\infty,a]}(\alpha)$ and $\text{length}_{[a,\infty)}(\alpha)$, where $a \in \mathbb{R}$ and length takes values in $[0, \infty]$.

Q2 (Angle)
Let $\alpha_i\colon I_i \to \mathbb{R}^n$, $i \in \{1, 2\}$, be two regular, continuously differentiable curves with $\alpha_1(t_1) = \alpha_2(t_2)$. The angle of intersection between α_1 and α_2 at t_1 and t_2 respectively is the angle a, which satisfies $0 \le a \le \pi$ and

$$\cos a = \frac{\alpha_1'(t_1) \cdot \alpha_2'(t_2)}{||\alpha_1'(t_1)||\ ||\alpha_2'(t_2)||}.$$

(a) Show that the angle of intersection is invariant under continuously differentiable (C^1) positive reparametrisations of α_1 and α_2.
(b) How does the angle change if one of the curves is changed by a C^1 positive and the other by a C^1 negative reparametrisation?
(c) How does the angle change if both curves are changed by C^1 negative reparametrisations?

Q3 (Curvature and torsion)
Let $\alpha\colon \mathbb{R} \to \mathbb{R}^3$ be defined by

$$\alpha(t) = (e^t \cos t, e^t \sin t, e^t).$$

Find the orthonormal frame (T, N, B), curvature κ and torsion τ as functions of t.