Assignment 1

"Some notions from MATH2000 with a view towards geometry"

Due Thursday, 4 August, at 16:00 in the assignment box for MATH3405. The box is located on **Level 4** in the Mathematics (Priestley) building (67). It is number 035 in the **brown lot of boxes** (there are two lots, to find ours, turn right as you come from the stairs).

Please use a cover sheet!

Clearly state your assumptions and conclusions, and justify all steps in your work. Marks will be deducted for sloppy or incomplete working.

Q1 (Volume)
Let V be the volume of the region inside the sphere $x^2+y^2+z^2=4$ and outside the cylinder $x^2+y^2=1$.

(a) Write down two triple integrals giving V, one using cylindrical coordinates and one using spherical coordinates.

(b) Evaluate V using either of the above integrals.

Q2 (Surface area in $\mathbb{R}^3$)
Find the surface area of the part of the sphere $x^2+y^2+z^2=a^2$ lying between the planes $z=b$ and $z=c$, where $-a \leq b < c \leq a$. Verify that this is the same as the surface area of the cylinder $x^2+y^2=a^2$ lying between these planes. (You may write down the area of the cylinder without any calculation.)

Q3 (Surface area in $\mathbb{R}^2$)

(a) Let C be a simple, closed, piecewise smooth curve bounding the region D in the plane, i.e. $\partial D = C$. Apply Green's theorem to $F(x,y)=(-y,x)$ to obtain the formula:

$$\text{Area}(D) = \frac{1}{2}\int_C x\ \mathrm{d}y - y\ \mathrm{d}x,$$

where C is oriented anti-clockwise. Use a similar trick to obtain the identities:

$$\text{Area}(D) = \int_C x\ \mathrm{d}y = -\int_C y\ \mathrm{d}x.$$

(b) Find the area enclosed by the hypocycloid $x^{2/3}+y^{2/3}=a^{2/3}$, where a is a non-zero constant. (Hint: Parameterise the curve in an analogous falshion to the standard parameterisation of the circle, use the first formula above and persevere with the integration.)