Notes on hand evaluation

by Richard Cowan
October 1987.

My recent article on hand evaluation has started to generate quite a correspondence and just a little controversy. In response to the many questions and comments, I have prepared a few notes. These are set out in a question and answer style. Hopefully, those who struggled a little with the mathematics in my article will find these notes more to their taste.

Some of my answers are incomplete, but time limitations have prevented me from further explorations at this stage. The published study itself took about 12 years of part-time effort. Alas, bridge research has always been over-shadowed by my normal scientific commitments.

Q. What made you embark on the study?

A. I was actually seeking a justification for the 4-3-2-1 scheme. I was completely unconvinced by existing arguments that purported to justify 4-3-2-1. The scheme is simple, it seems to work and it has had 60 years of empirical testing, so surely it could be supported by a convincing theory.

In many respects my study is now the best justification of 4-3-2-1. It shows that, amongst evaluation schemes that simply score points for individual honour cards (additive schemes), 4-3-2-1 is quite reasonable on jointly flat hands. Initially I looked for the optimal weightings for the top 5 honours as an academic exercise, but when the weightings (which were not strictly integers) turned out to be so close to the proportions 5:4:3:2:1, I was quite excited. I found it interesting that 4-3-2-1 was OK, but not the best, and that the best was equally simple.

Q: So how would you summarise the conclusions for a non-mathematical audience? The paper was, after all, published in a specialist mathematical journal.

A. There are some types of hands where one places great dependence on high-card point count. These are, of course, the flat hands. A partnership which devotes most of its bidding space below game level to shape description (a policy which I support strongly), is totally dependent on point count when <u>both</u> partners are flat. On the more distributional hands one depends less on point count. In this situation one assesses controls, loser-counts, lengths, shortnesses, texture of the long suits, the role of the short suits and "fit". The mildly distributional hands without a trump fit but with reasonable "communication", say 5-3-3-2 opposite 2-4-2-5, use point count as a guide tempered by long-suit texture and the knowledge that fewer points are needed to bid game in NT.

So, I believe that the point-count system should be tailored to perform best where it is most needed, on the commonly occurring jointly-flat hands. A good bidding system and the other evaluation tools take over when distribution is evident. So the study focusses on jointly flat hands. To evaluate the worth of honour cards in this restricted context, one needs to evaluate the number of tricks that each jointly flat combination will produce, averaged over all possible ways that the remaining 26 cards are distributed. But how do we evaluate the average number of tricks for even one example, say

(a) North: AJxx Kxx xxx Axx
South: Qx ATxx KJTx Qxx.

One needs to state a playing strategy for each potential lead and each contingency that evolves as the tricks are played, and use this to calculate the average number of tricks made. One must also average over the two cases where North or South are dummy. If we had the analytic powers to perform this task, we might come up with a statement like this: the pair of hands will produce 6 tricks 1% of the time, 7 tricks 5%, 8 tricks 40%, 9 tricks 42% and 10 tricks 12%. (I have made up these numbers for illustrative purposes.). Thus the average number of tricks would be 8.59. Let us call this the worth of the two hands. Another (easier) example is

(b) North: KQxx QTx Axx Kxx
South: JTx KJxx Kxx Axx.

In this case, we can calculate the worth as 9.75, assuming that the leader leads his longest suit or chooses from equally long suits. Under this assumption there is a 36.27% chance of a Spade or Heart lead and hence 10 tricks. On the remaining 63.73%, a Diamond or Club is led. Suppose West is on lead and leads Clubs. Declarer's best line is to hold-up on the first trick, winning the second round of Clubs. He/she forces out the Spade Ace (say). At this stage we can list the outcomes of the hand for each possible split of the Clubs and each location of the two missing Aces. If West has both Aces, declarer makes 9, 8, 7 or 6 tricks when the Clubs split 4-3, 5-2, 6-1 or 7-0 respectively. If West has AH and Clubs are 4-3, 9 tricks result. Otherwise, 10 tricks are assured. Formal probability calculations show that declarer makes 6 tricks with chance 0.05%, 7 with 0.83%, 8 with 4.69%, 9 with 13.17% and 10 with chance 81.27%, thus yielding the average of 9.75.

Whilst we can calculate the worth of some hands, we cannot do so for each of the 5.8×10^{20} jointly flat holdings. So I have taken the approach of analysing the average number of tricks for each possible suit holding (see Table 3 in the paper) and adding these to give a rating to full hands. I call this rating "trick taking potential", or TTP. Consulting Table 3, you will see that (a) and (b) have TTPs of 8.41 and 10.00 respectively compared with their worths of 8.59 (based on my guessed outcome) and 9.75. The TTP rating is not the same number as the worth, but I feel that it is a useful number on which to base a study of point count. It is sometimes up, sometimes down on the worth. I believe that TTP and worth are reasonably close provided declarer retains reasonable control of the play.

The paper shows that, within the context of additive schemes, the TTP of two flat hands is best estimated (prior to dummy's disclosure) by a 5-4-3-2-1 scheme for the top 5 honours. The paper arrives at this conclusion by analysing all jointly flat holdings that a partnership might have and using some substantial mathematics. The study makes no assumptions about the locations of key cards with the opponents, nor about suit splits. The TTP of a pair of hands is a measure of trick generating potential averaged over all possible splits and key-card locations.

Perhaps more importantly, the paper establishes a method for exploring the issues of hand evaluation. When time permits, I propose to use the basic data of the paper (Table 3) and the mathematical method (Appendix) to evaluate honour combinations in the style of Culbertson or, more recently, Steen. I anticipate big improvements in predictive accuracy over simple additive schemes. In due course I will be able to quantify well known maxims: honours in combination are better than honours separated; honours in long suits are more valuable than those in short suits. (Here we have only 4, 3 or 2 card suits, but there is a clear gradation of worth.)

I published in the Journal of The Royal Statistical Society partly because the paper was of a suitable standard for a refereed scientific journal, partly because I wished to place the mathematical details on record, but mainly because I wanted a journal held in libraries throughout the world. Every university library will subscribe to this journal, but (sadly) it is hard to find even municipal libraries which keep the well-known bridge magazines.

Q. It is hard to break a lifetime habit. Counting with 5-4-3-2-1 will be tricky.

A. Yes, but one correspondent, Danny Kleinman, has suggested a simple way. Score initially using the familiar 4-3-2-1, then count the <u>number</u> of honours A through T. Add the two entities together. He remarks that one can think of adding quality of honours to quantity of honours to yield my point-count score.

Q. It may not be easy for the bridge community at large to comprehend the mathematical arguments.

A. Yes, so perhaps a paraphrase will be useful. Recall that Table 3 gives, for each relevant suit holding, the average number of tricks that the optimal playing strategy will produce. For example, Axxx/KJx averages 2.87. (In the paper I often speak of this figure as the TTP of the suit holding, but I will now use lower case (ttp) when referring merely to suits and upper case when referring to full hands. Also, note that I have revised a number of entries in Table 3 recently, but with negligible impact on the results.) Table 3 (my revised version) can be summarised to enhance comparisons between honours. In so doing, one can demonstrate informally why the mathematics has found the 5-4-3-2-1 scheme optimal.

Since the 5-4-3-2-1 system implies that Q plus J equals A, let us take all joint holdings in Table 3 containing Q and J but not A and compare with those containing A but not Q nor J. Table A lists such comparisons, firstly when they are supported by the remaining honours (K and/or T), then when they are unsupported. For example, all the 4-4 holdings of Table 3 containing Q+J supported by K have average ttp of 2.73 whilst those containing the A supported by the K have average ttp of 2.68. (Note that we do not imply that the said honours are held in the one hand; they are distributed over both hands of the holding.)

	4-4	4-3	3-3	4-2	3-2	2-2
KQJT	3.00	3.00	2.00	3.00	2.00	1.00
AKT	2.88	2.54	2.03	2.09	2.02	2.00
KQJ	2.73	2.47	2.00	2.04	2.00	1.00
AK	2.68	2.36	2.00	2.00	2.00	2.00
QJT	1.85	1.69	1.00	1.18	1.00	0.00
AT	1.79	1.39	1.00	1.01	1.00	1.00
QJ	1.41	1.06	0.60	0.62	0.52	0.00
A	1.68	1.36	1.00	1.00	1.00	1.00

Table A: Summaries of Table 3 for those combinations where Q plus J can be compared with A. Tables show ttp averaged over all relevant holdings. Note that 4-3, 4-2 and 3-2 include holdings of type 3-4, 2-4 and 2-3 respectively.

The interpretation of Table A is heavily influenced by the knowledge (see Table 2 of the paper) that the 4-4, 4-3, 3-3, 4-2, 3-2 and 2-2 holdings occur with relative frequencies of 15.47%, 36.12%, 17.25%, 16.18%, 12.92% and 2.06% (after counting 3-4, 2-4 and 2-3 too). In particular, only about 2% of the suits played in this context are 2-2, so the last column has little weight. Thus we see that in the more prevalent parts of Table A, Q plus J is often more powerful than A, this being tempered by a clear advantage to A in the "rare" 2-2 case and in the situation where neither Q+J nor A is supported by the remaining honours. Certainly there is less support for the classical 4-3-2-1 scheme which has the A as clearly superior to Q plus J.

The exercise can be repeated to compare, say, J+T with Q. This is shown in Table B.

	4-4	4-3	3-3	4-2	3-2	2-2
AKJT	3.43	3.32	2.50	3.14	2.46	2.00
AKQ	3.68	3.36	3.00	3.00	3.00	2.00
AJT	2.27	1.95	1.30	1.57	1.23	1.00
AQ	2.21	1.87	1.50	1.50	1.50	1.17
KJT	2.09	1.80	1.22	1.32	1.15	0.50
KQ	1.92	1.63	1.21	1.20	1.15	1.00
JT	0.89	0.60	0.00	0.11	0.00	0.00
Q	0.92	0.60	0.24	0.16	0.14	0.00

Table B: Summaries of Table 3 for those cases which permit comparison of J+T with Q. In the more prevalent parts of the table, there is little to choose between J+T and Q.

At the risk of boring you, Tables C, D, E and F show comparisons of A+T vs K+J, Q+T vs K, K+T vs A and K+T vs Q+J respectively.

	4-4	4-3	3-3	4-2	3-2	2-2
AQT	2.55	2.18	1.66	1.78	1.61	1.17
KQJ	2.73	2.47	2.00	2.04	2.00	1.00
AT	1.79	1.39	1.00	1.01	1.00	1.00
KJ	1.58	1.28	0.85	0.83	0.79	0.50

Table C: A+T versus K+J. In the prevalent parts of this table, we see that A+T is only barely as good as K+J.

	4-4	4-3	3-3	4-2	3-2	2-2
AQJT	3.37	3.22	2.50	3.02	2.31	1.50
AKJ	3.19	2.83	2.38	2.49	2.36	2.00
AQT	2.55	2.18	1.66	1.78	1.61	1.17
AK	2.68	2.36	2.00	2.00	2.00	2.00
QJT	1.85	1.69	1.00	1.18	1.00	0.00
KJ	1.58	1.28	0.85	0.83	0.79	0.50
QT	1.17	0.78	0.30	0.28	0.18	0.00
K	1.21	0.87	0.50	0.50	0.50	0.50

Table D: Q+T versus K. Little to choose between them! Q+T responds better to the addition of J or A&&J whilst K is better unsupported or with the incremental A.

	4-4	4-3	3-3	4-2	3-2	2-2
KQJT	3.00	3.00	2.00	3.00	2.00	1.00
AQJ	2.90	2.58	2.08	2.22	2.05	1.50
KQT	2.35	2.00	1.52	1.57	1.46	1.00
AQ	2.21	1.87	1.50	1.50	1.50	1.17
KJT	2.07	1.80	1.22	1.32	1.15	0.50
AJ	1.89	1.56	1.10	1.13	1.08	1.00
KT	1.36	1.01	0.55	0.57	0.54	0.50
A	1.68	1.36	1.00	1.00	1.00	1.00

Table E: K+T versus A. In the prevalent cases, K+T is better than A when other honours are present, otherwise worse.

	4-4	4-3	3-3	4-2	3-2	2-2
AKT	2.88	2.54	2.03	2.09	2.02	2.00
AQJ	2.90	2.58	2.08	2.22	2.05	1.50
KT	1.36	1.01	0.55	0.57	0.54	0.50
QJ	1.41	1.06	0.62	0.62	0.52	0.00

Table F: K+T versus Q+J. The latter has a slight edge!

A similar comparison between A+J and K+Q (Table G) shows approximate equality in worth, but this is a property of both 4-3-2-1 and 5-4-3-2-1.

	4-4	4-3	3-3	4-2	3-2	2-2
AJT	2.27	1.95	1.30	1.57	1.23	1.00
KQT	2.35	2.00	1.52	1.57	1.46	1.00
AJ	1.89	1.56	1.10	1.13	1.08	1.00
KQ	1.92	1.63	1.21	1.20	1.15	1.00

Table G: A+J versus K+Q, with the latter a shade in front.

So, the pattern is clear. For the suit holdings in Table 3, 5-4-3-2-1 stands up to direct comparisons between honour combinations that would be deemed equivalent under that scheme.

Q. Can we see similar tables for honours that would be equivalent under a 4-3-2-1 scheme?

A. Here they are. The only cases to consider are A vs K+J (Table H) and K vs Q+J (Table I).

	4-4	4-3	3-3	4-2	3-2	2-2
AQT	2.55	2.18	1.66	1.78	1.61	1.17
KQJT	3.00	3.00	2.00	3.00	2.00	1.00
AQ	2.21	1.87	1.50	1.50	1.50	1.17
KQJ	2.73	2.47	2.00	2.04	2.00	1.00
AT	1.79	1.39	1.00	1.01	1.00	1.00
KJT	2.07	1.80	1.22	1.32	1.15	0.50
A	1.68	1.36	1.00	1.00	1.00	1.00
KJ	1.58	1.28	0.85	0.83	0.79	0.50

Table H: A comparison of A and K+J.

	4-4	4-3	3-3	4-2	3-2	2-2
AKT	2.88	2.54	2.03	2.09	2.02	2.00
AQJT	3.37	3.22	2.50	3.02	2.31	1.50
AK	2.68	2.36	2.00	2.00	2.00	2.00
AQJ	2.90	2.58	2.08	2.22	2.05	1.50
KT	1.36	1.01	0.55	0.57	0.54	0.50
QJT	1.85	1.69	1.00	1.18	1.00	0.00
K	1.21	0.87	0.50	0.50	0.50	0.50
QJ	1.41	1.06	0.60	0.62	0.52	0.00

Table I: A comparison of K and Q+J.

These tables show that Q+J in a suit (distributed randomly between the two hands) is clearly superior to K, whilst K+J is generally better than A. The tables also show why this is so. In each case, the pair of lower honours improve markedly when (by chance) other honours join it. By contrast, the high honour improves less markedly with this "stuffing". Without the "stuffing" and in the "rare" 2-2 case, the higher honours have greater trick taking potential than my new scheme would suggest, but in the bulk of cases, 5-4-3-2-1 out-performs 4-3-2-1.

Q. In those 2-2 cases, and some of the 3-2, the A has a dominant role.

A. Yes, but we must be careful to distinguish between its trick-generation role and its "control" role. In the tables we are seeing only its worth in trick generation! Later we shall discuss its additional controlling role.

Q. Is it not unfair to use 4-3-2-1 without half points for Tens, when 5-4-3-2-1 explicitly values Tens?

A. Yes, of course, but in the paper I show that, if one decides not to value Tens, the optimal weightings for Ace through Jack are still very close to the proportions 5:4:3:2. There is some deterioration in predictive accuracy of TTP rating if we use 5-4-3-2-0 rather than 5-4-3-2-1. My measure of predictive accuracy rises (using the revised data) from .2413 to .3065. On the other hand, the measure drops from .3418 for 4-3-2-1 to .2894 for 4-3-2-1-.5. (The lower the measure the better the predictive accuracy of TTP from adding up points.) Of course, the conclusions from tables such as H and I are the same whether tens are scored as zero or one half.

Q. Could we see your measure "average squared discrepancy" for the Stayman scheme 4.5-3-2-1-0 and the old "four aces" scheme 6-4-2-1-0?

A. The Stayman figure is worse at .3846, whilst "four aces" is worse still at .4909. With .5 for Tens they both improve, to .3371 and .4552 respectively.

Q. What do these numbers mean?

A. When I analyse a scheme (eg Work's 4-3-2-1-0) I derive a formula which best estimates TTP. For every scheme there is a different formula [eg. the revised (2) and (3) of the paper are

$$TTP = -1.53+.282P_C \quad \text{and} \quad TTP = -0.29+.361P_W$$

respectively]. The 4-3-2-1 formula estimates TTP for hand (a) as 8.37 compared with the true TTP of 8.41. The 5-4-3-2-1 formula estimates 8.34. For hand (b), we estimate TTP as 9.10 using 4-3-2-1 and 9.19 using 5-4-3-2-1, compared with a true TTP of 10.00.

The best estimates of TTP vary about my actual TTP rating because they are based on an additive point-count scheme which uses information available before dummy is exposed whilst TTP itself is a more accurate calculation using the detailed information of both hands. My "average squared discrepancy" is a measure of the spread of estimates about the true TTP. (The statistician calls its square-root the "standard deviation" and one can show that about 2/3 of the TTP estimates will lie within one standard deviation of the actual TTP. So, 2/3 of the 4-3-2-1-0 estimates will lie within 0.58 of my TTP ratings, whilst 2/3 of the 5-4-3-2-1 estimates will lie within 0.49 of that number.)

Q. In the preparation of Table 3, you assume that all x-cards are low, but on some occasions they will include useful nines and eights. Take the example AJx/Txx. With a 3/8 chance, the 9 will be included, and it makes quite a difference.

A. This point has worried me a bit over the years. I made the assumption to treat x's as insignificant to shorten the calculations, which were extremely lengthy in any case. Most players would not wish to value the 9 formally during the bidding phase, but occasionally it plays a key role during hand play, both in bolstering the minor honours such as J and T and because it can be left as the dominant card amongst the x's.

Of course, if I had been lazier I might have neglected Tens as well, treating all Tens as insignificant x's. I would now be faced with assessing the consequences of that decision (just as now I must assess the effects of neglecting Nines). Because my Table 3 did <u>not</u> neglect Tens, I am able to tabulate the <u>marginal</u> ttp of Tens in various situations. I can see whether the addition of a Ten is best with Jacks, Queens, Kings or Aces. Intuitively, we expect it to have more effect in bolstering the neighbouring honours, J and Q, than in bolstering the distant honours K and A. Table J sets out the <u>marginal ttp</u> of Tens with various combinations of the higher honours. By this I mean the <u>average</u> increment in ttp when the T is added to the combinations. There is variation about this average. For example, AQxx/xxx has ttp of 1.87 improving to 2.05 with AQxx/Txx or to 2.36 with AQTx/xxx. Axxx/Qxx improves from 1.87 to 2.18 (Axxx/QTX) or to 2.16 (ATxx/Qxx). Qxxx/Axx gains from 1.87 to either 2.12 (Qxxx/ATx) or 2.22 (QTxx/Axx). The average of these gains, weighted to account for the frequencies, yields 0.31 as shown in Table J.

	4-4	4-3	3-3	4-2	3-2	2-2
AKQJ	0.00	0.00	0.00	0.00	0.00	0.00
AKQ	0.17	0.22	0.00	0.41	0.00	0.00
AKJ	0.24	0.49	0.12	0.65	0.10	0.00
AK	0.20	0.18	0.03	0.09	0.02	0.00
AQJ	0.47	0.66	0.42	0.80	0.26	0.00
AQ	0.34	0.31	0.16	0.28	0.11	0.00
AJ	0.38	0.39	0.20	0.44	0.15	0.00
A	0.11	0.03	0.00	0.01	0.00	0.00
KQJ	0.27	0.53	0.00	0.96	0.00	0.00
KQ	0.43	0.37	0.31	0.37	0.31	0.00
KJ	0.49	0.52	0.37	0.49	0.36	0.00
K	0.15	0.14	0.05	0.07	0.04	0.00
QJ	0.44	0.63	0.40	0.56	0.48	0.00
Q	0.25	0.18	0.06	0.12	0.04	0.00
J	0.13	0.19	0.00	0.10	0.00	0.00
none	0.00	0.00	0.00	0.00	0.00	0.00

<u>Table J</u>: The marginal ttp of Tens with combinations of other honours.

The average gains in Table J vary a lot, but it is clear that Tens have greatest marginal impact when the suit holding contains Jacks or Queens. Indeed, despite some interesting cases where Q gains more than J, there is a trend. J benefits most, next Q, then K with least benefit going to A. Thus, had I neglected Tens, I would have introduced a bias in favour of the <u>higher</u> honours. We can extrapolate. By neglecting Nines in our current study, we have (if anything) undervalued T and J and overvalued K and A. Had we included 9, 8, 7, ... in an exhaustive way, we would have concluded that even 5-4-3-2-1 <u>overvalues</u> Aces! Of course, there are diminishing returns with these low cards. I feel that neglecting Nines introduces much less bias than a policy of neglecting Tens.

I would like to push Table 3 further to allow for 9, 8, ..., but with my current work commitments I would require lots of help (like 20 mathematically able people, each prepared to do 50 cases so that each result is double checked). To give you a flavour of the complexity, I work some examples.

Consider AKQ/xxxx which Table 3 rates as 3.355 (rounded to 3.36) on the assumption that x's are low. Four tricks result on the 35.5% chance that the suit splits 3-3. But AKQ/9xxx has the added chance (3.23%) that JT will be doubleton, so its ttp would be 3.388. Here, the 9 has a role as a "dominant x" trick. So with more diligence, I might have scored AKQ/xxxx as 3.388 times 3/8 plus 3.355 times 5/8, or 3.367 (rounded to 3.37). This is not worth worrying about. Moreover, I do not think that any examples where the 9 plays a "dominant x" role will produce significant changes in Table 3.

My concerns are with the other case, where 9 can bolster the J and T by either setting up finesse positions or by assisting to "force" higher honours. It is here that I would appreciate assistance, for the strategic analysis and probability calculations are very time-consuming. As a test of your tenacity and skill (which may be better than my own), try KQJx/xxx which Table 3 rates as 2.678 (rounded to 2.68) on the basis that x's are low. To get you started note that, with probability 0.200, one might have either KQJ9/xxx, KQJ8/9xx, KQJ7/98x or KQJ6/987, whereupon the T might fall or be finessed on the third round, yielding 2.84 (why?). But the story doesn't end there; try analysing the optimal playing line and ttp for KQJ7/9xx or KQJx/87x, etc. They all vary in subtle ways. My guess is that the 2.68 will move up to about 2.73.

Perhaps the exercise of evaluating Nines is not worthwhile, but every now and then an example pops up where the 9 makes a large difference. Your example, AJx/Txx, jumps from 1.33 to 1.76 with a 9, a 3/8 chance, so a more correct score for AJx/Txx in my Table 3 would be 1.76 times 3/8 plus 1.33 times 5/8, or 1.49. An occasional big improver makes little difference to the conclusions of my paper, but remember if there is <u>any</u> bias, the scales would tip further to 5-4-3-2-1 in preference to 4-3-2-1.

Q. There remains the issue of control. You assume that declarer retains control, so that suits can be played to their potential. This is your basis for the use of a hand's <u>TTP rating</u> instead of the more elusive <u>worth</u> .

A. Yes. Jeff Rubens has pointed me to some interesting comments in Squire's "The Theory of Bidding". On page 6, Squire remarks that the Ace is often over-valued in no-trumps, and that it can be under-valued only when its immediate worth is as a control. He gives two deals of jointly flat hands, deal A has 26 points, deal B 25. The hands do not appear atypical, nor do they have any important nines or eights, etc.. He gives deal A, Axxx/Axx/Kxx/Kxx with Qxx/Qxxx/Axx/Axx, little chance of 9 tricks but rates B, Ax/Qxx/Qxxx/QJxx with KQx/AJxx/JTx/KTx, practically sure. Squire remarks that "obviously the Aces are overvalued in deal A". My new system scores A as 34 points and B as 38.

His example to illustrate the vital role of Aces for control is, unfortunately, not one of the jointly flat deals: Ax/Ax/AQTxxx/Axx with Kxx/QT9x/Jx/98xx. He says "A Spade is led against 3NT. Declarer clears Diamonds with the loss of the king and the defence clears Spades. Declarer cashes 9 tricks. But, had he the KQJ of Clubs instead of the ace he would have gone down, losing three Spades, one Diamond and one Club. The value of that ace as control was enormous."

Point taken, but really one should look at the hands assuming randomized E/W holdings. Give declarer the KQJ of Clubs instead of A, as Squire suggests. A Spade lead is not certain (only about a 46% chance for a leader who leads his longest suit, or chooses from equally long suits). The loss of KD is not certain. Spades, if led, may be 4-4. If 6-2 or worse, they may lack the vital AC as entry. So, the circumstances where declarer prefers to hold A of Clubs instead of KQJ arise in a minority of deals. It is easy to construct scenarios where KQJ yields more tricks than Axx (as well it should). Of course, if we make Squire's hand flat (by transferring 2 Diamond x's to Spades and Hearts, say), it is played in 2NT with Club Axx and 3NT with Club KQJ, my TTP ratings being 8.58 and 9.58 respectively.

But I do not wish to belittle the importance of Aces in control, for I think that control is the crucial point in my assumption that <u>TTP rating</u> is a good measure of <u>worth</u>. Whilst I think that my studies to date are a useful contribution to the theory of bridge, they have not come to terms with the type of hand where declarer loses control. Because the fragile 2-2 holding affects only 8.24% of jointly flat hands, because the chances of the most dangerous lead are less than many people realise and because there are good chances of friendly suit splits in cases like 3-2 and 4-2, I suspect that loss of control is not as prevalent as our "bad memories" suggest. (Our experiences are clouded by memories of all the NT contracts involving non-flat hands.)

It has been suggested that I should have negative ttp values for the 2-2 holdings in Table 3, and perhaps some of the 3-2. Someone suggested the xx-xx should have ttp of minus "5 and a bit", but this is too severe. True, it is likely that the opponents will run 5 tricks. This forces declarer to find 3 discards in his stronger suits, thus depriving them of trick potential. I do not know precisely how to score this, but it is probably best to give xx-xx (and like 2-2 holdings) a ttp of about -2 to reflect the loss in ttp in the other suits. I shall try these modifications shortly. Those readers who have contacted me <u>since</u> receiving my paper will be kept posted of developments. Of course, anybody who wishes to participate in (a) my exercise with the 9's, (b) this dilemma about control or (c) a proposed empirical test which assesses the closeness of TTP and worth, will be welcomed.

Q. What do you mean by an empirical test?

A. I am not sure, but many people have suggested a dose of empiricism to test my theory. Perhaps the following would be a start. I generate (randomly by computer) n jointly flat hands of appropriate strength. I ask 20 experts to give each holding a worth (eg. 9.25), or (since calculating worth is extremely difficult) they could be asked to <u>rank</u> the list of holdings. I could assess the concordance of the consensus of the rankings with my TTP rating and with the estimate of TTP using 5-4-3-2-1 and 4-3-2-1 schemes. Amusingly, I could also assess the concordance of the experts. It could be fun!