SAMPLE HONOURS ESSAY TOPICS

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My research area is geometric analysis, which employs analytic tools such as partial differential equations to study problems in geometry and topology. I have worked on geometric flows such as Ricci flow and mean curvature flow, (moduli) space of Riemannian metrics, and geometry problems that arise in mathematical general relativity.

I look forward to supervising honours student(s). You honours essay topic could include:

(a) **Existence of "nice" metrics**. A fundamental question in differential geometry is that if a given a manifold M carries a "nice" Riemannian metric. For example, an n-sphere quipped with the standard round sphere is nice in the sense that its curvatures are all the same positive constant. Two research topics in this direction are:
- **Ricci flow**[1]: Here one deforms a metric g by $\partial_t g = -2\mathrm{Ric}(g)$, where $\mathrm{Ric}(g)$ is the Ricci curvature of g. Ricci flow is an efficient way to deform the metric into a "nice" one known as the Einstein metric. However, Ricci flow is nonlinear and tends to develop singularities in finite time, and singularity analysis poses a fundamental problem in the theory.
- **Yamabe problem**: Here one seeks the existence of metric with constant scalar curvature in a given conformal class on a given manifold, e.g. the round metric on an n-sphere. This question can be formulated as an elliptic PDE, or can be approached by parabolic methods.

(b) **(Moduli) space of metrics**. Given a manifold M, denote by $\mathcal{M}$ the space of all Riemannian metrics on M. Then we can study the properties of the space $\mathcal{M}$. For example, is there a tangent space to $\mathcal{M}$ at a point (i.e. a metric)? If so, what is it? Or, is the space $\mathcal{M}$ path-connected? What about its higher homotopy groups? If two metrics g, g' are diffeomorphic, then (M,g) and (M,g') are geometrically indistinguishable. Thus, denote by $\mathfrak{D}$ the space of all diffeormphisms of M, we also study the moduli space $\mathcal{M}/\mathfrak{D}$.

(c) **Zoo of submanifolds**. Classical differential geometry studies curves and surfaces which are submanifolds of the flat Euclidean $\mathbb{R}^3$ (cf. MATH3968). One can generalise to other ambient space (M,g) and study submanifolds of special properties such as *minimal* (zero mean curvature) and *constant mean curvature* (CMC) submanifolds. For example, Which CMC submanifolds exist in a given (M,g)? How many are there? Can we classify them? In this context, one can also study *mean curvature flow*.

(d) **Mathematical general relativity**. The space-time of our universe is studied using Riemannian geometry (space) and Lorentzian geometry (space-time) and contains many fascinating topics. To name a few: the mathematics of black holes is related to that of minimal surfaces in a space-time; the Positive Mass Theorem and its connection to scalar curvature; the Riemannian Penrose Inequality can be proved using the inverse mean curvature flow.

If you have other topics related to geometry, topology, or analysis in mind, please do not hesitate to email me so we could discuss the possibilities.

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[1]Ricci flow has been successful in proving Thurston's Geometrisation Conjecture of three manifolds and the Poincar/'e Conjecture, one of the "million-dollar maths problems".