Limits for functions of two (or more) variables

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that

$$\lim_{x \to 0} f(x, 0) = \frac{1}{2}; \qquad \lim_{y \to 0} f(0, y) = \frac{1}{2}.$$

What can we say about $\lim_{(x,y) \to (0,0)} f(x, y)$?

(a) We don't have enough information to say anything.

(b) We don't know if the limit exists, but if it does exist, it must be $\frac{1}{2}$.

(c) The limit exists and is equal to $\frac{1}{2}$.

(d) I don't understand the question.

Make-up lecture on Wednesday's material

- Today, 3pm, right here in this room
- Lecture by Prof. Tolman; slides available from her website (link on Piazza course information page).
- Students from all sections welcome.
- 100% optional.

Finding limits of continuous functions

Consider

$$f(x,y,z) = \frac{\sqrt{y}}{x^2 - y^2 + z^2}.$$

Find

$$\lim_{(x,y,z)\to(0,1,0)} f(x,y,z).$$

(a) -1

(b) 0

(c) I've got $\epsilon > 0$, now I'm looking for δ, and I need more time.

(d) I don't know how to start.

Practice with partial derivatives

Let $f(x, y) = \sin(3x + xy)$. Calculate $f_x(x, y)$.

(a) $\cos(3x + xy)$

(b) $(3 + y)\sin(3x + xy)$

(c) $(3 + y)\cos(3x + xy)$

(d) $x\cos(3x + xy)$.

Practice with higher partial derivatives

Let $f(x, y) = \sin(3x + xy)$. Calculate $f_{xy}(x, y)$.

(a) $-(3 + y)x \sin(3x + xy) + \cos(3x + xy)$

(b) $(3 + y)x \sin(3x + xy) + \cos(3x + xy)$

(c) $-(3 + y)x \sin(3 + xy)$

(d) $(3 + y)x \sin(3 + xy) + x \cos(3x + xy)$