

MATH 402 Review for November 12–16

Topics: Omega points, omega triangles, angle of parallelism, quadrilaterals and triangles in hyperbolic geometry.

These were covered in lecture, on worksheet 6, and in the project (7.4). This material also appears in Homework 10.

- 1. Recall from last week:** Recall the definition of a line being limiting parallel to another line through a point P . Remember that we showed that the choice of P doesn't matter, and that the notion of being limiting parallel gives an equivalence relation (as long as we're careful about the lines being limiting parallel "at the same end".)
- 2. Things to know about omega points and omega triangles**
 - (a) Know the definition of an omega points; know that it is really a *set of lines*, not a point, but that it behaves like a point in a number of ways.
 - We can talk about an omega point Ω *lying on a line* ℓ ; hence we can talk about two lines *intersecting at an omega point* (which means that they're limiting parallel to each other), and we can also talk about the *unique line* $\overrightarrow{P\Omega}$ through a point P and an omega point Ω .
 - (b) An *omega triangle* consists of two rays and one segment determined by two ordinary points and one omega point which are not collinear.
 - (c) Make sure you know the statement of **Pasch's Axiom for Omega Triangles, Parts I and II** (it's a theorem, not an axiom, despite the name!).
 - (d) State the **Exterior Angle Theorem for Omega Triangles**, and use it to prove that the angle sum of an omega triangle is always less than 180° .
 - (e) State the **SA Congruence Theorem for Omega Triangles**, and use it to prove the **AA Congruence Theorem for Omega Triangles**.
- 3. Things to know about the angle of parallelism**
 - (a) Recall the definition of the angle of parallelism (given a line ℓ , a point P not on ℓ , and a line m which is limiting parallel to ℓ through P).
 - (b) Use properties of omega triangles to prove that the angle depends only on the distance h from the point P to the line ℓ . This means that it makes sense to denote the angle simply by $a(h)$, rather than by writing something like $a(\ell, P)$.
 - (c) Use properties of omega triangles to prove that if $h < h'$, then $a(h) > a(h')$, and in particular, if $h \neq h'$ then $a(h) \neq a(h')$.
- 4. Things to know about quadrilaterals and triangles**
 - (a) Recall the definition of *Saccheri quadrilaterals* and *Lambert quadrilaterals*. Know that you can divide a Saccheri quadrilateral in half to get two Lambert quadrilaterals, and that you can reflect a Lambert quadrilateral to get a Saccheri quadrilateral.
 - (b) Use this fact together with the fact that the summit angles of a Saccheri quadrilateral are always congruent and acute to prove that the fourth angle (the non-right angle) of a Lambert quadrilateral is also acute.
 - (c) Know that the angle sum of a hyperbolic right triangle is less than 180° , and know how to use this to prove that the angle sum of any hyperbolic triangle is less than 180° .

- (d) Know the definition of the *defect* of a triangle or quadrilateral. Know that it is additive: if you cut a triangle into a triangle and a quadrilateral, the defect of the larger triangle is the sum of the defects of the smaller pieces.
- (e) Know that **AAA congruence** holds in hyperbolic geometry. Note that this might seem kind of weird!

Practice Questions

1. Take a triangle and cut it up into three smaller triangles in different ways. Prove that the defect of the original triangle is the sum of the defects of the smaller triangles.
2. Take two Saccheri quadrilaterals with congruent bases and congruent sides. Prove that they have congruent summits and congruent summit angles.
3. Let ℓ be a line, P a point not on ℓ , and m a line through P which is limiting parallel to ℓ . Prove that if you move P along m towards the omega point, the perpendicular distance from P to ℓ decreases. (Use properties of angle of parallelism.)