<u>Math 1931</u> – <u>Spec. Stud. Program</u>

Chaos and Dynamical Systems

Welcome!

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Carslaw 615, Consultation: Wed. 9am

I - Introduction / Motivation

Who invented Calculus?
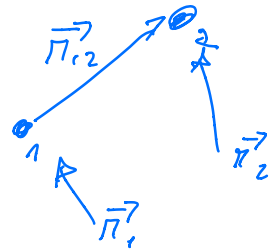
Gottfried Leibniz   discovered

Isaac Newton

When? 17th Century, 1642 - 1726

- Motion
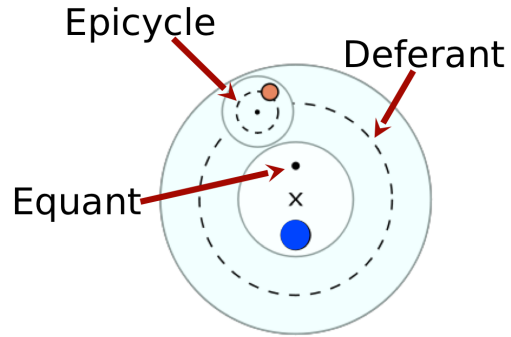- Gravity $\longrightarrow$ $F_G = -\dfrac{G\, m\, M}{|\vec{n}_{12}|^2}\, \hat{n}_{12}$
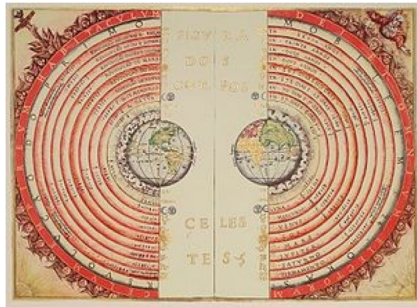- Classical Mechanics

$$\vec{F} = m\vec{a} = m\dfrac{d\vec{v}}{dt} = m\dfrac{d^2 \vec{n}}{dt^2}$$
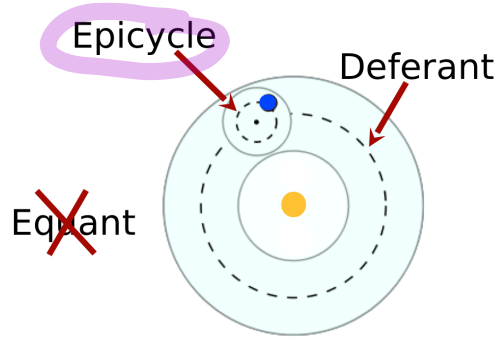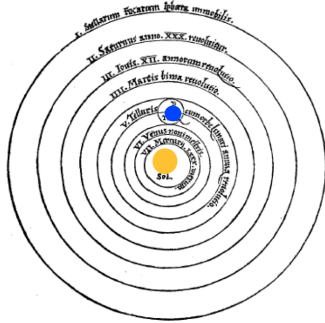
Cosmology (before Newton)
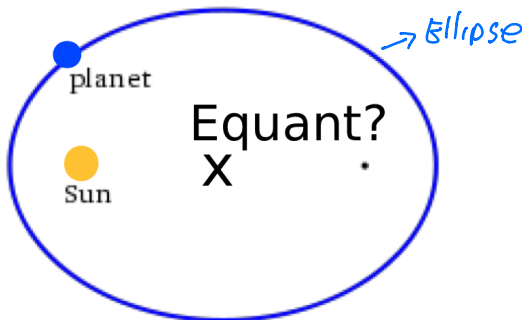## and the movement of celestial bodies

Ptolemaic system (Ancient Times -> 16 century)

Epicycle

Deferant

Equant

Copernican System (16 century)

Epicycle

Deferant

Equant

Keplerian System

Ellipse

planet

Equant?
X

Sun

What was Newton's innovation?

Halley visits Newton in Cambridge (August 1684):

*"Dr. Halley asked him what he thought the curve would be that would be described by the Planets supposing the force of attraction towards the Sun to be reciprocal to the square of their distance from it. Sir Isaac Newton replied immediately that it would be an Ellipsis, the Dr. struck with joy and amazement and aksed him how he knew it, Why saith he I have **calculated** it..."*

Classical (newtonian) Mechanics

$$\vec{F}_G \qquad \vec{F}_G$$
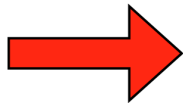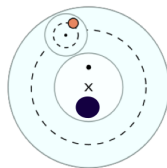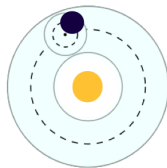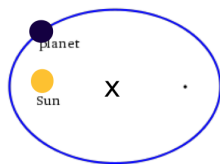
$$\vec{F}_G = \frac{-GmM}{r^2}\hat{r}$$

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a} \quad (2^{nd}\ law)$$

$$\vec{F}_G$$

Newton's recipe:

1 Find all forces acting on the body

2 - Use 2nd law of mechanics to compute movement

Def.: Are rules that describe how a system
    evolves in time.

Example:

$$m \frac{d^2 x}{dt^2} = -k x$$

x differential

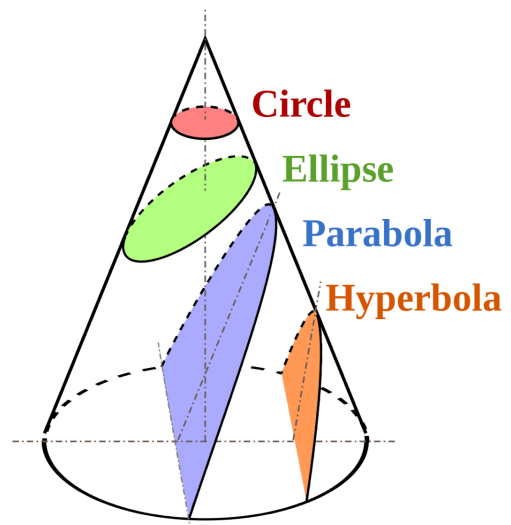   equation

The curve is the solution $x(t)$

      ⌐→ We call it ==a trajectory==

2 bodies interacting through
    Newton's gravitation

⟱

All curves/trajectories
are conical sections!

Circle

Ellipse

Parabola

Hyperbola

Implications of classical Mech. (determinism)

initial condition

"We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes."
— Pierre Simon Laplace (1795)

↳ perfect prediction

Implications of a simplistic Newtonian Science:

1- The goal of fundamenta Science (Physics) is to reveal what the fundamental forces of nature are.

2- Once we know the forces acting on a body (the dyn. syst.) We can predict the future.

Goal of these 4 lectures:

Show that 1. and 2. are wrong!

How?

1. There are many interesting/
surprising things that appear when
we compute solutions of .D.S.

2- Chaos limits predictability.