THE UNIVERSITY OF SYDNEY
MATH3063

Semester 1 2019

COURSE INFORMATION
MATH3063
NONLINEAR ORDINARY DIFFERENTIAL EQUATIONS WITH APPLICATIONS

Purpose of course: To introduce the qualitative theory of systems of nonlinear ordinary differential equations and to apply this theory to various problems in mathematical biology.

Summary of learning outcomes: At the end of the unit you will be able to:

- explain the principle of linear approximations to nonlinear systems and use this to analyse system behaviour close to steady states;
- synthesise graphical information from nullclines and flow to construct qualitative phase plane solutions to problems in nonlinear systems;
- demonstrate knowledge of the theory of existence and uniqueness of solutions of ordinary differential equations;
- interpret model results and evaluate and explain the limitations of models in representing real systems;
- demonstrate a broad understanding of the role of basic bifurcations in nonlinear systems and evaluate the effect of parameter variation on observed model behaviour;
- apply mathematical theory in novel and diverse applications.

These learning outcomes will be assessed in examinations, assignments and class quizzes. Achieving these learning outcomes will contribute to acquiring the university's Graduate Attributes, particularly depth of disciplinary expertise, critical thinking and problem solving, written communication skills, inventiveness and interdisciplinary effectiveness.

A week-by-week description of the content of unit is given on the last page of the unit of study outline.

This unit contributes towards the requirement of 24 cp of MATH3XXX units for the Mathematics major in the old courriculum and as a selective in the third year of the Mathematics major in the new curriculum.

All assignmments must be submitted electronically through the Canvas Learning Management System. Assignments written using TEX, LATEX, Word or any other software for text production will be put through similarity detection software (Turnitin). Scanned handwritten assignments will also be critically assessed for evidence of plagiarism. Any serious questions about academic integrity will be addressed using standard academic procedures of the University of Sydney, under the relevant policies, procedures and guidelines.

Please make sure that you complete the University's compulsory module on Academic Integrity if you have not done so already. A link to the new Canvas version of this module will be published on 25 February, 2019.

Lecturers: Dr Ian Lizarraga, Carslaw room 489. Email: `ian.lozarraga@sydney.edu.au`
Professor Mary Myerscough, Carslaw room 626. Email: `mary.myerscough@sydney.edu.au`

Attendance at lectures is not required but strongly recommended. As lecturers may choose to use blackboard presentations which are a standard part of the discipline culture in mathematics, lecture recordings will be of limited use. Past experience suggests that non-attendance a t lectures and tutorials can have an adverse impact of students' performance.

Tutorials start in Week 2. Rolls will be kept. This not only provides a record of student attendance, but help tutors to learn your name quickly.

Tutorial sheets and solutions will be posted on the unit web page. Please bring your copy to tutorials as no sheets will be handed out. You are expected to prepare for tutorials by revising the lectures and doing the tutorial work in advance.

Assessment The final mark will be made up of the following components.

: Examination (*70%*)
: Assignments, one due in Week 4 (*5%*) and one due in Week 10 (*10%*).
: Class tests to be held in a lecture in Week 7 (*10%*)and a lecture in Week 12 (*5%*).

Solutions to selected tutorial exercises and to assignments will be posted on the web as they become available.

Additional information including information about special considerations etc is available in the Senior Mathematics Handbook, which can be downloaded from the Senior Mathematics page `http://www.maths.usyd.edu.au/u/UG/SM/`

Ordinary Differential Equations and Biomathematics Course

: **Week 1** First order equations: Mathematical models–exponential and logistic growth; definition of linear and nonliner, autonomous and non-autonomous. Phase portraits, equilibria, stability (using phase portraits), linear stability, linearisation for single first order equations.

: **Week 2** Bifurcation in single, first order equations, including the expential and logistic models. Harvesting, both constant rate and constant effort. Population catastrophes. Bifurcation diagrams. Hysteresis.

: **Week 3** Introduction to predator-prey models, specifically the Lotka-Volterra model. Nonlinear models cannot be solved explicitly, hence the need for new mathematical tools. Introduction of the phase plane; nullclines, flows, sketching solutions. Phase plane of the Lotka-Volterra equations motivates the need for more information.

: **Week 4** Nonlinear systems and linearisation. Solving linear systems. Classification of the behaviour of linear systems (nodes, focuses, saddles, centres, etc). Phase planes of linear systems. The Jacobian matrix and classifying behaviour using its trace and determinant.

: **Week 5** Phase portraits and linear stability analysis of a variety of nonlinear systems (lots of examples). Existence and uniqueness of solutions. Different types of stability.

: **Week 6** Lotka-Volterra equations using linear analysis and phase planes. Harvesting the Lotka-Volterra equations. Structural instability. Other predator-prey systems. Models for ecological competition, mutualism.

: **Week7** Models for the spread of disease. Definition of an epidemic. Basic SIR model. Critical population sizes. Vaccination effects. What happens as $t \to \infty$. SIS and SIRS models, crisscross infections and STDs.

: **Week 8** Lyapunov stability. Finding and using Lyapunov functions. Sketch of Lyapunov theorems.

: **Week 9** First integrals, Hamiltonian systems and gradient systems. Definition of first integral. The Lotka-Volterra equations as an example of a Hamiltonian system. Conservative systems. nonlinear pendulum, Duffing equation, the Van der Pol oscillator.

: **Week 10** Limit cycles: definition, stability analysis, phase protraits. Biological examples (mainly computational).

: **Week 11** Bifurcation in systems of two first order ODEs. Statement of the Hopf bifurcation theorem. Creation of limit cycles. The Brusselator model. Predator-prey and epidemiological models with Hopf bifurcations.

: **Week 12** Fitzhugh-Nagumo equations, relaxation oscillations and excitable media