Short Answers: 2009 MATH1003 examination

1(a) $\frac{1}{4}\ln(1/3)$

1(b) $x(t) = \frac{8}{11}e^{8t} + \frac{3}{11}e^{-3t}$

1(c) $\pi^2/4$

2(a) $\int \ln x \, dx = x\ln x - x + C$

2(b) Upper Riemann sum $= \ln(n!)$ using $\Delta x = 1$.
Thus $\ln(n!) > \int_1^n \ln x \, dx$.

2(c) From part (b), $\ln(n!) > n\ln n - n + 1$,
so that $n! > \exp(n\ln n - n + 1) = e(n/e)^n$.

3(a) $y_P = 6x^5 - 30x^3 + 45x$.

3(b) $y = w + y_P = A\cos 2x + B\sin 2x + 6x^5 - 30x^3 + 45x$

3(c) 1st order DE for v: $v' + P(x)v = y_0R(x)$
The integrating factor $r(x) = \exp\int P(x)\,dx$.
Thus $v(x) = 1/r(x) \int r(x)y_0(x)R(x)\,dx$.
Finally, $y' - (y_0'/y_0)y = 1/(y_0r(x)) \int r(x)y_0(x)R(x)\,dx$.