THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Assignment 2

MATH4403: Asymptotic methods and perturbation theory — Semester 1, 2017

Lecturer: Sharon Stephen

This assignment is due by **11:59pm Saturday 27th May 2017**, via Turnitin. A PDF copy of your answers must be uploaded in the Learning Management System (Blackboard). Please submit only a PDF document (scan or convert other formats). It should include your name and SID. It is your responsibility to preview each page of your assignment after uploading to ensure each page is included in correct order and is legible (not sideways or upside down) before confirming your submission. The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions. External students may email their assignment to the lecturer.

This assignment is worth 25% of your final assessment for this course. Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas.

Answer all three questions.

1. Consider $I(N) = \displaystyle\int_P^Q z^2 \exp\{iN(z^3+3z)\}\,dz$ as $N \to \infty$.

(a) Let $f(z) = i(z^3+3z) = u + iv$, for real functions u and v. Sketch the level curves of u and v, indicating the saddle points and the directions of steepest decent.

Hence, use the method of steepest descents to determine the leading order terms in the asymptotic expansions of the following integrals as $N \to \infty$.

(b) $I_1(N) = \displaystyle\int_{1-i}^{1+i} z^2 \exp\{iN(z^3+3z)\}\,dz$

(c) $I_2(N) = \displaystyle\int_{-1+i}^{1+i} z^2 \exp\{iN(z^3+3z)\}\,dz$

2. Determine the leading order outer approximation and the leading order inner approximation to the solution of the differential equation

$$\epsilon y'' + x^{1/2}y' + y = 0 \quad 0 \le x \le 1,$$

satisfying the boundary conditions $y(0) = 0$ and $y(1) = 1$.

You may leave the inner solution in terms of an integral and the gamma function, $\Gamma(z)$, where $\Gamma(z) = \displaystyle\int_0^\infty u^{z-1}e^{-u}\,du$.

3. Use the method of multiple scales to determine a first-order uniform expansion for the solution of the following equations where $\epsilon \ll 1$. [Hint: work in complex form.]

$$\ddot{u} + u = \epsilon(1 - u^2)\dot{u}.$$

Your solution should be in terms of constants a_0 and β_0 such that $u(0) = a_0 \cos \beta_0$.